ANALYSIS AND DESIGN OF PILES SUBJECTED TO AXIAL AND LATERAL LOADS

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CERTIFICATE

This is to certify that the work presented in this thesis has been carried out by Mr. N. G. Rajasekhar under my supervision and has not been submitted elsewhere for a degree.

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CONTENTS

NOTATION

A =	= constant ;
A'	= constant ;
B	= constant ;
B'	= constant ;
b	= $(c-1)/L$;
C	= constant ;
C'	= constant ;
c	= ratio of the tip diameter to the top diameter of uniformly tapered circular cross-section pile ;
D	= constant ;
D'	= constant ;
D_o	= diameter of pile at the top ;
d'	= non-dimensional parameter ;
E	= Young's Modulus of Elasticity of pile material;
E_s	= Soil Modulus for a pile of width B ;
F	= Factor of Safety ;
I_o	= moment of Inertia of pile at the top ;
$I(x)$	= moment of Inertia of pile at any depth x from the top ;
K_o, K_1	= coefficients of soil modulus variation ;
K_p	= coefficient of passive earth pressure ;
L	= length of pile ;
L_1	= the depth of upper zone ;

M = bending moment ;

M_1 = bending moment due to lateral load Q_t for pile with free-free end conditions ;

M_2 = bending moment due to lateral moment M_t for pile with free-free end conditions ;

M_1' = non-dimensional moment coefficient for free-free pile only P and Q_t acting at the top ;

M_2' = non-dimensional moment coefficient for free-free pile only P and M_t acting at the top ;

M_{1f} = dimensional bending moment for fixed-free pile ;

M'_{1f} = non-dimensional bending moment for fixed-free pile ;

M_t = lateral moment coming at the top of the pile ;

M_{tD} = non-dimensional moment at the top ;

N = Number of samples tested.

P = axial force acting at the top of pile ;

P_f = Probability of soil reaching ultimate resistance.

$P(x)$ = axial force at any depth x from the top ;

p = soil reaction per unit length ;

Q_{1f} = non-dimensional shear force at the top for elasto-plastic soil ;

Q_t = lateral load acting at the top of the pile ;

Q_{tD} = non-dimensional lateral load at the top ;

R = relative stiffness factor ;

r_o = non-dimensional parameter ;

S_d = Design shear strength ;

S_f = random variable representing shear strength ;

$\bar{S}_f$ = mean shear strength of the test series ;

S_{ft} = True in-situ shear strength ;

s = best estimate of standard deviation of population ;

t = variable of Student's t-distribution function.

t = coefficient of ultimate soil resistance ;

U = $\bar{P}R^2/EI_o$ = non-dimensional parameter ;

U_{cr} = non-dimensional critical load ;

v = non-dimensional parameter ;

x = co-ordinate distance from the top ;

Y = deflection coefficient for layered soil ;

y = deflection coefficient for elasto-plastic soil ;

first subscript 1 = when only P and Q_t act on top of free-free pile ;

first subscript 2 = when only P and M_t act on top of free-free pile ;

2nd subscript f = refers to fixed-free pile ;

2nd subscript l = refers to lower zone ;

2nd subscript u = refers to upper zone ;

superscript (') = non-dimensional ;

Z = non-dimensional depth coefficient ;

Z = Standardized normal variable.

Z' = non-dimensional maximum depth coefficient ;

Z_1 = non-dimensional depth of upper zone ;

α, γ = parameters in the characteristic equations ;

λ, η, σ, σ', τ, τ' ψ' = non-dimensional parameters ;

ABSTRACT

Flexural response of piles to externally applied loads is one of the complex soil-structure interaction problems. For a realistic analysis of the pile problem it is necessary to consider the proper boundary conditions, axial force variation, variation of moment of inertia of the pile and load-deflection relationship of the pile. It has been concluded from the literature review (Chapter 2) that the analysis of an individual pile can be carried out at working loads using the concept of soil modulus whereas at larger loads the plastic resistance developed in the top region has to be taken into account.

In Chapter 3, closed form solutions are obtained for axially and laterally loaded tapered piles in layered soils for different boundary conditions. In Chapter 4 closed form solutions are obtained for individual pile in elasto-plastic soil.

In Chapter 5, after reviewing the existing literature on the probabilistic approaches to soil mechanics problems, the need for considering the uncertainties involved has been stressed. Also an initiation has been made to bring about the influence of factor of safety, number of samples and variance on the probability of failure.

CHAPTER 1

INTRODUCTION

In foundation engineering the study of flexural behaviour of piles forms one of the complex soil-structure interaction problems. For a realistic analysis of the flexural behaviour of a pile, it is necessary to know the soil pressure distribution at the interface between the pile and the soil, which inturn is governed by a host of parameters. Hence if one has to take all the parameters into account the analysis becomes extremely complex. However with the experience and the data available from the field tests and laboratory experiments, some simplifying assumptions can be made to arrive at simple analytical solutions for many cases corresponding to field situations.

On the basis of study of the work done by various investigators in this field one can have a broad idea about the qualitative nature of contact pressure at the interface. The systematic studies carried out by Matlock and Ripperger (1957, 1957a), Peck and Davisson (1958, 1962), Broms (1964a, 1964b) and Ke'risal (1965) show that the flexural analysis of piles can be made based on the concept of soil modulus at working loads. According to this

concept, the soil pressure, p, is assumed to be directly proportional to the deflection at that point. The works of Biot (1937) and Vesic'(1961) deal with the validity of this assumption in the analysis of beams on elastic foundation. From the analysis of extensive field and laboratory data, Broms (1964 a, 1964 b) has shown, that the concept is quite valid for analysing the laterally loaded piles at working loads. However, if the soil in the top region is quite loose and does not take much load, then there will be large deflection or rotation due to the applied loads, and the problem becomes much more complicated due to non-linear characteristics of the soil and this needs modification in applying the concept of soil modulus.

From the available literature, it is seen that the flexural behaviour of the pile, in general, is governed by the soil properties in the top region upto a depth of 10 to 15 pile diameters from ground level. As the properties of the top region may be different from those of the lower region it is very important to obtain solution for flexural response of pile in a layered soil. This, also brings out the drawbacks of the existing literature, where in either constant or linearly varying soil modulus is assumed. Recent studies of Davisson and Gill (1963), Reddy and Valsangkar (1968, 1971), Basudhar

(1971) deal with the problem of flexural analysis of uniform cross-section pile, at working loads, in layered soil. Relatively few solutions are available for the flexural analysis of piles whose moment of inertia varies with depth because of mathematical complexities. Mostof the solutions use numerical techniques for the analysis and with this in view in Chapter 3 generalized solutions are presented for laterally and axially loaded tapered piles in layered soil. Numerical results are presented exhaustively, bringing out qualitatively the influence of soil modulus variation, layering and axial force variation.

Engineers, in addition to understanding the behaviour at working loads, are also interested in the ultimate carrying capacity of the soil -structure system. In case of laterally loaded piles, to determine the maximum load carrying capacity,it is assumed that,the soil fails all along the length of pile or that the section of the pile fails, depending upon the length and boundary conditions of the pile at top (Broms, 1964a, 1964b). However many field studies indicate that in reality the situation lies somewhat intermediate, in the sense, only some part of the soil at the top may go into plastic condition, but the depth of plastic zone may not be sufficient to induce excessive bending moments, so that pile failure takes place. So, for a realistic analysis both elastic and plastic properties have to be considered,

as both states occur along the embedded length of the pile. Studies of Wagner (1953), Matlock, Ripperger and Fitzgibbon (1956), Matlock and Ripperger (1957a), Mori (1964), Reddy and Valsangkar (1970), M.B. Roy (1970) and Basudhar (1971) support this view. Recently, Domenico Lalli (1971) has presented approximate solutions for the behaviour of piles of variable cross-sectional properties and head angular restraint, in bi-linear soils under both lateral and axial loads by large deflection Matrix method. In the above case the results are presented for a pile of particular dimensions and varying lateral loads, axial loads and head angular restraint in dimensional forms.

In Chapter 4, analytical solutions in closed form are presented for a tapered pile in elasto-plastic soil with generalized loading . As the solutions are obtained in closed form these can be used for checking the convergence of numerical methods used for solving such problems. Also the results can be directly used in field situations wherein most of the assumptions made in the analysis are satisfied. Even in cases wherein the axial force variation and moment of inertia variation may be different than the one assumed in the analysis, the solutions can be used to get fairly good estimate of flexural response. Numerical data has been obtained comprehensively, to bring about

the influence of various parameters qualitatively. Results are presented in non-dimensional form so that they can be useful for design purposes.

It is well known that the soil properties vary from point to point even in an apparently uniform soil, due to the random nature of the soil. As such in recent times many authors have considered the uncertainties in the soil properties in the analysis of soil mechanics and foundation engineering problems.

In chapter 5 an initiation has been made for taking into account the uncertainties involved by adopting the probabilistic approach in the ultimate load analysis of laterally loaded piles. The chapter also contains a review of all the available literature concerning probabilistic methods as applied to soil mechanics and foundation engineering problems.

CHAPTER 2

REVIEW OF LITERATURE

2.1 GENERAL

This Chapter deals with a review of the experimental and analytical investigations pertaining to flexural behaviour of piles under the action of lateral or lateral and axial loads. In the review of the experimental investigations, attention is given mainly to the recent studies on instrumented piles, which have added to the clear understanding of this complex soil-structure interaction problem. Also detailed review is available in the works of Davisson (1960), Valsangkar (1969) and Basudhar (1971). At the end of the Chapter, generalized conclusions are drawn, which form the basis for the assumptions made in the subsequent chapters on the analysis of piles.

2.2 EXPERIMENTAL INVESTIGATIONS

For the first time the contact pressure distribution at the interface was measured by Stobie (1930), while conducting experiments on behaviour of poles. The study brought out the fact, that the passive resistance offered by the pile is considerably more than the two-dimensional passive pressures obtained

by Rankine's theory. But the measurements were very crude.

Experimental investigations were conducted by Feagin (1937), Matsu (1939), Silts, Graves and Driscoll (1948) on uninstrumented piles. The results of these studies bring out the significant influence of repeated loading, batter of the pile and stiffness of the pile as affecting the flexural behaviour.

For the first time, systematic studies to measure the soil pressure by using strain gauges were conducted by Loos and Breth (1949) on model steel vertical and batter piles. The soil modulus was assumed to vary linearly for the analysis of test results and the correlation between the predicted values and observed values was seen to be good. The results also revealed that, increasing the length of the pile beyond a certain limit had practically no effect on resisting lateral loads.

McCammon and Ascherman (1953) presented the results of the tests conducted on long hollow piles of length of about 170 feet, with an overhang of 80 feet embedded in plastic clay. The results indicate that the clay acted as an elastic medium and the point of maximum moment occured very near to the ground surface for both free and restrained piles.

Gleser (1953) has shown from the field tests, that the presence of vibrating operations near the piles increases the deflections.

Barber (1953), while discussing the test results of Gleser, reported the results of the tests carried out on monotube piles with or without axial loads. The test piles were embedded 80 to 90 feet in organic clay. From these test results it was concluded that, the deflections get considerably magnified in the presence of axial loads in addition to lateral loads.

Mason and Bishop (1954) presented the results of the tests conducted on a full scale pile embedded in earth fill and subjected to lateral loads. The pile was held in vertical position and the sand around it was well compacted. Mathematical analysis was presented based on soil modulus concept and the results of theoretical analysis and observed values were reasonably in good agreement. Mason (1956) by continuing the studies at the same site with repetitive loading on the piles, reported that the repetition of loading increases the deflection by about 20 per cent.

McNulty (1956) carried out tests on Raymond Standard piles and on wooden piles. Three unreinforced concrete Raymond piles were embedded in a soil deposit upto 14, 22 and 26 feet. The soil at the site consisted of 3 feet of sandy clay at the top below which there was fine silty sand. The water table was at the ground surface. It was interesting to note that in the absence of axial load unreinforced Raymond pile failed at relatively low loads as practically there was no flexural resistance offered. However, when the tests were conducted with axial loads, the piles showed greater lateral resistance as the compressive stresses from the axial load kept the tensile stresses

caused by the lateral load to a minimum.

For the first time most exhaustive and systematic experimental investigations, both in laboratory and field were conducted by Matlock and Ripperger (1956, 1957 a, 1957 b).

Matlock, Ripperger and Fitzgibbon (1956) conducted a series of tests on a steel pile with 12.75 inches outside diameter and 0.5 inch wall thickness. The test pile was driven to a depth of 42 ft. in a fresh water lake deposit. The soil reactions along the embedded length of the pile were measured by strain gauges. The test site consisted of predominantly clay, with intermediate sand layers. At first instance two series of tests were made, one of the static load and the other of cyclic load.

The results of the static load tests indicate that the soil resistance was plastic for large lateral loads for upper 4 to 6 feet of soil. It was also observed that, for cyclic load test, the soil resistance in the upper 14 feet of soil was plastic and the soil resistance was nearly elastic below 14 feet. This considerable decrease in the soil resistance in the upper region compared to the static case can be attributed to the considerable remoulding of the soil due to cyclic loading. From the analysis of the test results it was also seen that the soil modulus variation was linear in the upper region and constant in the lower region. Matlock and Ripperger (1957 a) continued the tests at the same site by varying the soil condition at the top 10 feet of the soil deposit. This test brought

out the beneficial effect of placing sand or gravel in the top region around the pile in resisting lateral loads.

Matlock and Ripperger (1957 b) performed studies on model aluminium piles embedded in gelatin. Photoelastic studies were performed on the laterally loaded piles. It was seen from these studies that, major part of the load was dissipated within a distance of 3 pile diameters from the face of the pile. It was also observed that at higher loads plastic resistance developed within a distance of 1.5 pile diameter, from the face of the pile. It was also concluded from these tests that, for a given lateral load the deflections of the pile were independent of pile diameter.

Davisson (1960) reports the lateral load tests carried out by California Division of Highways in 1958, on reinforced concrete piers cast in bored holes. The soil consisted of loose sand underlain by dense sand. Electrical strain gauges were used to measure the deflections along the length of the pile. Tests were also conducted on partially embedded piles with an overhang of 8 feet, which showed considerable increase in lateral deflections at the top.

Peck and Davisson (1962) reported the tests conducted on steel pile of 54.3 feet in length and embedded in silty soil, with the pile tip resting on hard rock. The analysis of the data based on linearly varying soil modulus showed good agreement between predicted and observed values.

Mori (1964) conducted full scale lateral load tests on steel pipe piles embedded in loose to dense sand. The deflections at the top were measured under the action of lateral loads. The analysis based on the assumption that the soil near the ground surface reached ultimate resistance showed that, the measured values were in aggrement with the expected values.

Kérisel (1965) described the results of tests conducted on steel caisson embedded in clay. Values of displacements, slopes, moments and pressures were measured along the embedded length of the pile by using electrical strain gauges. The point of maximum bending moment was observed to be not at a greater depth from the top. Reese (1965) analysing the above results showed that the analysis can be carried out on the assumption of linearly varying sail modulus. He also showed that, the approximate expression given by him (Reese,1958) to determine the ultimate soil resistance at the top is reasonably valid.

Kubo (1965) reported the exhaustive results of the laboratory studies made on model steel piles. On the basis of the experimental results a non-linear soil pressure-deflection relationship which varies linearly with depth was proposed. It was also observed from the test results that the soil at the top goes into plastic region.

Milaan and Eousberg (1965) suggested an optical method which can be adopted for measuring the horizontal pressures coming on piles when subjected to lateral loads.

Prakash and Saran (1967) performed model tests on aluminium piles in cohesive soils. The tests were conducted on single and on 4 and 9-pile groups. It was concluded from the tests that the interference increased as the spacing decreased in the direction of horizontal loads.

Awad and Petrosovits (1969) presented the results of laboratory tests on model rigid piles embedded in cohesionless soils with different batters. The test results show that the variation of coefficient of soil modulus is considerably affected by batter of piles.

Davisson and Salley (1970) presented a relationship between the behaviour of a single and group of laterally loaded piles from the tests conducted at Arkansas River Navigation Project. They observed for a single pile (a) a triangular variation of subgrade reaction for initial loading (b) at low loads a sharp decrease in subgrade modulus with increasing deflection, and not much change in soil modulus at higher loads, (c) 100 cycles of load caused an increase in deflection to 1.7 times that observed for first cycle.

2.3 ANALYTICAL INVESTIGATIONS

In this section a comprehensive review of the analytical studies is presented. Most of the analytical studies reported in the literature treat the problem of flexural behaviour of pile as a beam or beam-column on elastic foundation. Hetenyi (1946) and Vlasav and Leonte'v (1966) have presented the solutions for the fundamental differential equation for several boundary conditions and for variable

moment of inertia and soil modulus.

Earlier works of Titze (1932), Ratzersforfer (1936, vide Davisson, 1960), Chang (1937) and Pender (1947) all deal with the analysis of the flexural behaviour problems on the basis of beam on elastic foundation concept.

Palmer and Thompson (1948) have proposed the finite difference approach to the problem of laterally loaded pile based on soil modulus concept and have shown that this approach can be used with ease even for the most complicated variation of soil modulus.

Subsequently, Barber (1953), Palmer and Brown (1954), Mason and Bishop (1954) and McClelland and Focht (1958) presented numerical solutions by making use of finite-difference approach, considering both constant and linearly varying soil moduli.

In 1955, Terzaghi in his classical paper on subgrade modulus proposed that for preloaded clays, the soil modulus can be taken as invarient with depth whereas for sands it can be assumed as linearly varying with depth.

Reese and Matlock (1956) proposed an iterative method of taking the soil modulus as the secant modulus in the non-linear range of the load deflection curve and make use of concept of beam on elastic foundation. For the solution they used the finite difference technique and presented the non-dimensional charts for deflection, moment and shear force values, for linearly varying soil modulus.

Davisson (1960) presented the solution for laterally and axially loaded pile with linear varying soil modulus and with invariant moment of inertia and axial force with depth, by electrical analog method.

Matlock and Ingram (1963) have presented a general method, for solving the problem of beam and beam-column on elastic foundation, based on finite-element mechanical models.

Davisson and Gill (1963) presented electrical analog solution for laterally loaded pile in a two-layered soil system. The stiffness of the surface layer is defined in terms of the underlying layer. The study clearly broughtout quantitatively the overwhelming influence of upper few feet of soil on the flexural behaviour of laterally loaded piles.

Mori (1964) presented solution for laterally loaded pile in elasto-plastic soil. Plastic resistance in the plastic region was assumed to be constant with depth. In the elastic zone the soil modulus was assumed to be constant with depth.

Broms (1964 a, 1964 b) presented, the "ultimate resistance" analysis of laterally loaded piles in both cohesive and cohesionless soils. It is assumed for laterally loaded pile the plastic resistance develops all along the length of the pile for rigid pile. Whereas for the case of flexible pile, the ultimate load carrying capacity is arrived at from the consideration of failure of the pile because of

development of plastic resistance in the top region of the soil. Based on this previously published data were analysed and good agreement was seen between the predicted and measured values. On the basis of comparison with the field and laboratory measurements he also showed that the concept of soil modulus is reasonably valid at working loads.

Spillars and Stoll (1964) presented analysis of laterally loaded pile by making use of Mindlin's theory for a point load in semi-infinite continuum. In the analysis the effect of plastic region was also taken into consideration.

Poulos (1971 a) presented the solution for horizontal displacement and rotation of a vertical pile subjected to lateral loading and moment, and situated in a homogenious, isotropic and semi-infinite elastic mass using Mindlin's theory approach. Numerical results are presented for a wide range of pile flexibilities and length to - diameter ratios, for both free -free and fixed-free cases. Comparisons between the elastic solutions and the corresponding solutions obtained from Winkler's theory show that, the latter considerably overestimates the displacement and rotation of the pile, but gives a very reasonable estimate of the moments in the pile. The elastic analysis is also extended to include the effect of local yield between the soil and the pile. Poulos (1971 b) also extended the analysis to group of piles. Based on the same approach recently, Poulos and Madhav (1971) presented the results for axially loaded piles and laterally loaded piles.

Matlock and Grubbes (1965) analysed the problem of laterally loaded pile in elasto-plastic soil. The non-linear nature of the deflection curve is considered in the analysis. Extensive numerical results are presented to bring about the effect of various parameters.

Reddy and Valsangkar (1966, 1967, 1968, 1969, 1970, 1971) made a comprehensive study on the flexural behaviour laterally and laterally and axially loaded pile problem. For many cases solutions have been given in closed form, series solution and also by energy method. The effect of variation of soil modulus, variation of moment of inertia of the pile, and variation of axial force has been presented in these works. Solutions are also presented for the problem of laterally loaded pile in both two-layered soil system and Elasto-plastic soil.

Roy (1970) considered the different stages, starting from fully elastic to fully plastic soil behaviour, and presented the solutions for laterally loaded pile problem.

Domenico Lalli (1971) presented the solutions for the problem of pile with generalized loading in non-linear soils by 'large deflection matrix methods'. The variation of cross-sectional properties of pile is considered and numerical results are presented using the proposed method for a particular case.

Basudhar (1971) obtained solutions for the laterally and axially loaded pile problem in series form and the results are presented in non-dimensional forms. The effect of variation of

axial force and soil modulus is considered and exhaustive numerical results are presented for layered soil, elasto-plastic soil and for a pile driven through a compressible strata.

2.4 GENERAL CONCLUSIONS

Based on the review of the experimental and analytical investigations certain generalized conclusions are drawn below.

(i) The flexural behaviour of a pile is governed by the properties of top region of the soil of about 10 to 15 diameters of the pile.

(ii) Coefficient of soil modulus and plastic resistance are considerably influenced by different factors, viz : Stiffness of the soil, remoulding, repetition of lateral load, vibrations during driving, consolidation coefficient and size of loaded area.

(iii) For a more realistic analysis of pile problem, it is necessary to consider both elastic and plastic state of the soil. Especially at higher loads it is known that the soil at the top goes into plastic region.

(iv) When in addition to lateral loads, axial load also is acting, the flexural capacity of the pile considerably decreases.

(v) The soil modulus concept is reasonably valid at working loads.

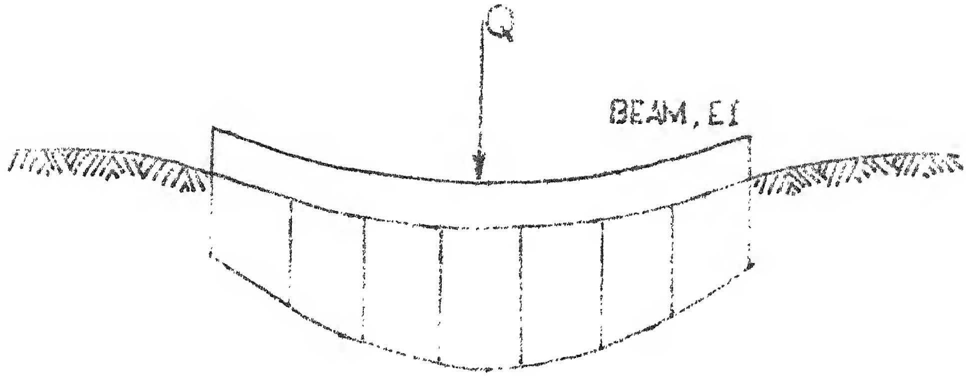
(vi) Boundary conditions play an important role in the flexural behaviour of pile.

(vii) Field tests have shown that Terzagni's (1955) recommendations are fairly valid for soil modulus variations.

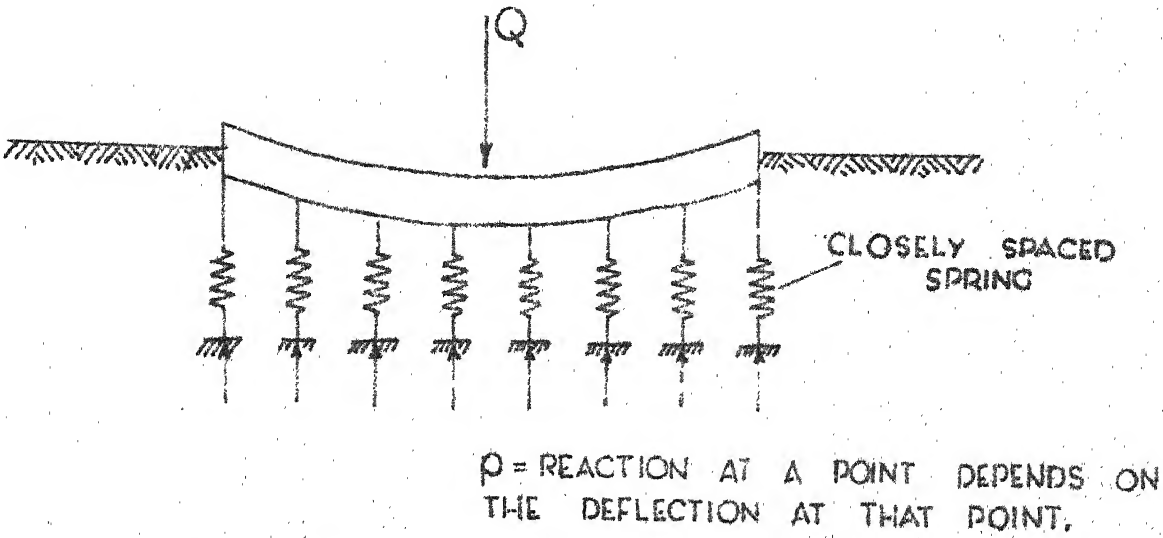
(viii) Brom's approach can be used for finding the ultimate carrying capacity of the pile with reasonable accuracy.

(ix) As the review shows that the behaviour is considerably influenced by a variety of parameters and as such a probabilistic rather than deterministic formulation will be an ideal one.

(x) From the tests of Poulos (1971 a) it can be concluded that the winkler's approach can be used with reasonably good accuracy to obtain the bending moment with depth for a pile problem, and the elasticity theory has to be used to get a fairly good estimate of deflection and slope.



(a) BEAM ON ELASTIC CONTINUUM.



(b) WINKLER MODEL

FIG. 2.1. - BEAMS ON ELASTIC CONTINUUM AND WINKLER MODEL

CHAPTER 3

GENERALIZED SOLUTION FOR LATERALLY AND AXIALLY LOADED TAPERED PILE IN LAYERED SOIL

3.1 GENERAL

The necessity of flexural analysis of axially and laterally loaded piles arises from the use of these piles in marine structures, oil drilling rigs, cranes and other similar structures wherein these piles have to resist large lateral and axial forces. It is well known that, when in addition to axial forces, the pile foundations have to resist large lateral forces, the flexural capacity of the pile foundations considerably decreases, because of "Beam-column effect". Field investigations of Arup (1944), Evans (1953), Barber (1953), Mcnulty (1956), Peck and Ireland (1961) and Francis et.al. (1961) support this view. Hetenyi (1946) has presented many analytical solutions for a beam-column resting on elastic foundation for different boundary conditions.

Eventhough the overwhelming effect of the upper layer on the flexural behaviour of piles was known longback, analytical studies were presented only recently by Davisson and Gill (1963) for the case of uniform cross-section pile in a layered soil with constant soil modulus in each layer. Recently Reddy and Valsangkar (1968) considered

the general variation of soil modulus in the form of a second degree polynomial and presented the solution for a pile of uniform cross-section embedded in a layered soil and subjected to lateral load only. Basudhar (1971) obtained the generalized solution for a pile of uniform cross-section, embedded in a layered soil, acted upon by a lateral load, moment and an axial load, considering the effect of variation of axial force along the embedded length of the pile.

In many instances step-tapered, uniformly tapered and stepped piles are used for transfer of load from superstructure to subsoil. In case of step-tapered piles, because of corrugated surface and the various steps, the load carried by the skin-friction component is considerable and as such these piles are used as friction piles (D' Appolonia and Hribar, 1963). Experimental studies by Mcnulty (1956) and, Peck and Ireland (1961) on Raymond standard and step-tapered piles show that reinforced piles have better resistance to lateral loads when compared to unreinforced piles.

Review of the literature also reveals that, the analytical studies taking into account variation of moment of inertia are very few. Reese and Gingbarg (1958) have proposed finite difference method for the analysis of laterally loaded piles with abrupt changes in moment of inertia and linearly varying soil modulus.

As the available literature confines to only uniform cross-sectioned piles in layered soil, analysis has been made in this chapter

to arrive at a generalized solution for the case of a tapered pile embedded in a layered soil and acted upon by generalized loading.

3.2 STATEMENT OF THE PROBLEM

Fig. 3.1 (a) shows a uniformly tapered pile with circular cross-section whose moment of inertia varies with fourth power of x. Fig. 3.1 (b) shows a free head tapered pile which is acted upon by a lateral load, Q_t, moment, M_t, and an axial load, P, at the top. Fig. 3.1 (c) shows schematically a fixed head pile, wherein it is assumed that the pile is free to translate and is restrained against rotation. Fig. 3.2 (a) and Fig. 3.2 (b) show the general representation of laterally loaded pile problem along with the possible complexities in the behaviour of the soil. In Fig. 3.2 (a) the form of springs, indicate, that the stiffness of the soil varies with depth and deflection of the pile. The St. Venant's body elements at the top indicate that there is some limit to the soil reaction which can be developed in the top region and the Newtonian dash-pots represent the time dependent visoous effects of the soil. It should also be noted that the repetition of load alters the behaviour of the soil pile system. However taking all these factors into account makes the problem involved mathematically, as such the analysis presented herein takes into account elastic nature of the soil, variation of moment of inertia and variation of axial force along the embedded length of the pile. Fig. 3.2 (b) shows the probable variation of soil reaction along the embedded length of the pile.

Fig. 3.3 (a) and Fig. 3.3 (b) depict the linear and non-linear soil pressure-deflection relationships respectively. Even if the p-y relationship is linear E_s may vary with depth and , hence a general relationship would be, $p = E_s(x)\ y$. If p-y relationship is not linear as shown in Fig. 3.3 (b), then the soil modulus will be a function of both x and y. Matlock and Reese (1956) suggested the use of the secant modulus and repeated application of the elastic theory to arrive at the solution of this non-linear problem.

Since the concept of soil modulus is an important factor in the following chapters a brief discussion of the same is given below.

Generally, for all soils, the value of coefficient of soil modulus will be smaller at ground surface than at some depth below the surface. According to Terzaghi (1955) the soil modulus is constant with depth for pre-loaded clays as shown in Fig. 3.4 (a). In many cases there may exist at the top a stiff crust due to dessication or some other mechanism. Then the probable variation of soil modulus is as shown in Fig. 3.4 (b). Fig. 3.4 (c) represents the probable variation of soil modulus for the case of a weak soil layer over a hard layer (Davisson and Gill 1963). For sands E_s is assumed to vary linearly with depth as shown in Fig. 3.4 (d). For normally loaded clays also variation of E_s is given by Fig. 3.4 (d) (Davisson, 1960).

Fig. 3.5 and Fig. 3.6 show a free-head pile and a fixed -head pile respectively, in a two layered soil with second order polynomial variation of soil modulus in each layer, which can be considered to

be the most general variation (Reese and Matlock, 1956). However, in the present work it is assumed that the soil modulus is constant in each layer, so as to have mathematically elegent solutions. As stated earlier, this case corresponds to the variations as depicted in Figs. 3.4 (a), (b) and (c).

As in the present case, the projected width of the pile varies with x, a brief discussion regarding the effect of width on coefficient of soil modulus is given.

Terzaghi (1955) has presented a detailed discussion regarding the effect of width on the coefficient of soil modulus. Fig. 3.7 shows a pile embedded in cohesive and cohesionless soils alongwith the pressure bulbs for piles of widths B and nB. For linear behaviour of soil and the same intensity of loading, the pressure bulb for a pile of width B for both clays and sands is as shown in Fig. 3.7 . Because of this the deflections for a pile of width nB will be n times the deflections for pile of width B. Thus the coefficient of soil modulus for a pile of width B is equal to the coefficient of soil modulus for a unit area multiplied by width B. Because of this fact, the soil pressure, p, per unit length of the pile remains invariant with depth (Davisson, 1960). However, the above reasoning is only valid for elastic behaviour of the soil and the effect of width is different in the non-linear range of p-y relationship.

The laboratory studies of Matlock and Ripperger (1957 b)

confirm Terzaghi's recommendations regarding the effect of width on the coefficient soil modulus. They also concluded that the deflections of a pile for a given lateral load should be the same regardless of pile diameter. Hence in the present chapter, taking the validity of Terzaghi's (1955) recommendations, analytical solutions are obtained for constant soil modulus. It should be noted, that the analysis presented herein should be used only for linear p-y relationship.

The variation of moment of inertia with depth x for the tapered pile shown in Fig. 3.8 (a) is given by :

$$I(x) = I_o(1 + bx)^4 \qquad \text{.... (3.2.1)}$$

in which I_o = moment of inertia at the top ; $b = \frac{c-1}{L}$

L = length of the pile,

and c = ratio of bottom diameter to top diameter.

Eventhough in the present analysis a continuous variation of moment of inertia is assumed, the same analysis can be used for finding the flexural response of stepped and step-tapered piles, as in general the steps at each section are small and one can approximate the pile by one which is uniformly tapered.

The variation of axial load depends on the distribution of skin friction which in-turn depends on the properties of the soil, pile etc. For clays it is a time dependent phenomenon whereas for sands it is invariant with time.

It should be noted that similar to contact pressure distribution at the interface the axial force distribution is governed by many factors and as such generalization regarding the distribution is not possible. As such in this work it is assumed to vary parabolically as shown in Fig. 3.8 (b). Thus the axial force at any depth x is given by :

$$P(x) = P(1+bx)^2 \qquad \ldots . (3.2.2)$$

in which

P = axial force at the top of the pile.

Therefore the axial force at the tip is given by :

$$\underset{\text{at } x=L}{P(x)} = Pc^2 \qquad \ldots . (3.2.3)$$

So the load transferred due to skin friction for this type of variation of axial force is equal to $P(1-c^2)$. For c = 0.6 (usual value) the load transferred due to skin friction is 64 percent of total load coming at top. This particular form has been chosen as this would lead to a mathematically simple analysis. However, it should be noted that, at axial loads corresponding to 0.3 to 0.5 U_{cr} the effect of variation of axial force on the flexural behaviour is practically negligible (Reddy and Valsangkar, 1968) and as such in these range of axial loads it is immaterial which form of axial force is assumed. Thus the assumed variation though being particular, the results can be used for the load ranges upto 0.5 U_{cr} without much errors.

3.3 ANALYSIS

The governing differential equations for the uniformly tapered pile shown in Fig. 3.8(a) wherein, constant soil modulus is assumed in each layer are given by (Hetenyi, 1946) :

$$\frac{d^2}{dx^2}\left[EI(x)\,\frac{d^2y_u}{dx^2}\right]+\frac{d}{dx}\left[P(x)\,\frac{dy_u}{dx}\right]+K_o y_u = 0 \quad 0\leq x\leq L_1 \quad \ldots\ldots (3.3.1)$$

$$\frac{d^2}{dx^2}\left[EI(x)\,\frac{d^2y_l}{dx^2}\right]+\frac{d}{dx}\left[P(x)\,\frac{dy_l}{dx}\right]+K_1 y_l = 0 \quad L_1\leq x\leq L \quad \ldots\ldots (3.3.2)$$

It is obvious from equations (3.3.1) and (3.3.2) that, even for the simplest case when the axial force remains constant with depth, it is difficult to obtain the analytical solution. However, for the particular axial force variation shown in Fig. 3.8, it will be shown herein that, it is possible to arrive at a closed form solution.

Substituting for $I(x)$ and $P(x)$ from equations (3.2.1) and (3.2.2) into equations (3.3.1) and (3.3.2), the above equations transform to :

$$\frac{d^2}{dx^2}\left[EI_o(1+bx)^4\,\frac{d^2y_u}{dx^2}\right]+\frac{d}{dx}\left[P(1+bx)^2\frac{dy_u}{dx}\right]+K_o y_u = 0 \quad \ldots\ldots (3.3.3)$$

$$0\leq x \leq L_1$$

$$\frac{d^2}{dx^2}\left[EI_o(1+bx)^4\,\frac{d^2y_l}{dx^2}\right]+\frac{d}{dx}\left[P(1+bx)^2\,\frac{dy_l}{dx}\right]+K_1 y_l = 0 \quad \ldots\ldots (3.3.4)$$

$$L_1\leq x \leq L$$

To fecilitate the numerical computations, equations(3.3.3) and (3.3.4) are transfermed into non-dimensional form by making use of the following non-dimensional parameters :

Depth coefficient $Z = \frac{x}{R}$ .... (3.3.5)

$$Z_1 = \frac{L_1}{R} \qquad \text{.... (3.3.6)}$$

and maximum depth coefficient; $Z' = \frac{L}{R}$ .... (3.3.7)

in which R = relative stiffness factor

In this work, it is taken as :

$$R = \sqrt[4]{EI_o/K_1} \qquad \text{.... (3.3.8)}$$

In the present analysis, since elastic theory is used, by assuming the validity of the principle of superposition the actions of (i) axial load, P and lateral load, Q_t, (ii) axial load, P and moment, M_t are considered separately for free head pile. Thus if y_1 and y_2 are deflections due to (i) and (ii) respectively, then the total deflection is given by :

$$y = y_1 + y_2 \qquad \text{.... (3.3.9)}$$

Similarly, the bending moment, M under the combined action of (i) and (ii) is given by :

$$M = M_1 + M_2 \qquad \text{.... (3.3.10)}$$

in which M_1 = B.M. due to (i)

M_2 = B.M. due to (ii) .

dimensional deflection and moment coefficients are defined as follows.

Case (i) Free-free end conditions pile, only Q_t and P acting at the top :

Deflection coefficient, $$Y_1' = \frac{y_1 EI_o}{Q_t R^3} \quad \text{.... (3.3.11)}$$

and Moment coefficient, $$M_1' = \frac{M_1}{Q_t R} \quad \text{.... (3.3.12)}$$

in which y_1 and M_1 are dimensional deflection and bending moment values respectively.

Case (ii) Free-free end conditions pile, only M_t and P acting at the top :

Deflection coefficient, $$Y_2' = \frac{y_2 EI_o}{M_t R^2} \; ; \quad \text{.... (3.3.13)}$$

and Moment coefficient, $$M_2' = \frac{M_2}{M_t} \quad \text{.... (3.3.14)}$$

Case (iii), Fixed-free end conditions pile, with Q_t and P acting at the top :

Deflection coefficient, $$Y_{1f}' = \frac{y_{1f} EI_o}{Q_t R^3} \; ; \quad \text{.... (3.3.15)}$$

and Moment coefficient, $$M_{1f}' = \frac{M_{1f} EI_o}{Q_t R} \quad \text{.... (3.3.16)}$$

in which y_{1f} = dimensional deflection and M_{1f} = dimensional bending

moment for fixed - free pile.

By making use of the above non-dimensional coefficients equations (3.3.3) and (3.3.4) reduce for the above case (i) to :

$$\frac{d^2}{dZ^2}\left[(1+d'Z)^4 \frac{d^2Y'_{1u}}{dZ^2}\right] + U\frac{d}{dZ}\left[(1+d'Z)^2\frac{dY'_{1u}}{dZ}\right] + r_o Y'_{1u} = 0$$

$$0 \leq Z \leq Z_1 \qquad \ldots\ldots (3.3.17)$$

and

$$\frac{d^2}{dZ^2}\left[(1+d'Z)^4 \frac{d^2Y'_{11}}{dZ^2}\right] + U\frac{d}{dZ}\left[(1+d'Z)^2\frac{dY'_{11}}{dZ}\right] + Y'_{11} = 0 \qquad \ldots\ldots (3.3.18)$$

$$Z_1 \leq Z \leq Z'$$

in which

$$d' = \frac{c-1}{(L/R)}$$

$$U = \frac{PR^2}{EI_o} \qquad \ldots\ldots (3.3.19)$$

and $$r_o = \frac{K_o}{EI_o}R^4$$

Since the above differential equations are of Euler-Couchy type these are transformed into equations with constant coefficients by the following substitutions, (Ince, 1956, Kamke 1959, Kosko 1965) :

$$(1+d'Z) = \eta \qquad \ldots\ldots (3.3.20)$$

and $$\eta = e^v \qquad \ldots\ldots (3.3.21)$$

Then equations (3.3.17) and (3.3.18) transform to :

$$\frac{d^4Y'_{1u}}{dv^4} + 2\frac{d^3Y'_{1u}}{dv^3} + (\psi'-1)\frac{d^2Y'_{1u}}{dv^2} + (\psi'-2)\frac{dY'_{1u}}{dv} + r_o\lambda Y'_{1u} = 0 \qquad \ldots. (3.3.22)$$

and

$$\frac{d^4Y'_{1l}}{dv^4} + 2\frac{d^3Y'_{1l}}{dv^3} + (\psi'-1)\frac{d^2Y'_{1l}}{dv^2} + (\psi'-2)\frac{dY'_{1l}}{dv} + \lambda Y'_{1l} = 0 \qquad \ldots. (3.3.23)$$

in which $\psi' = \dfrac{U}{(d')^2}$

and $\lambda = \dfrac{1}{(d')^4}$ .... (3.3.24)

Equations (3.3.22) and (3.3.23) are homogeneous ordinary linear differential equations with constant coefficients for which closed form solutions can be arrived at as follows :

Writing in the operator form $D = \dfrac{d}{dv}$ we have

$$[D^4+2D^3+(\psi'-1)D^2+(\psi'-2)D + r_o\lambda]\,Y'_{1u} = 0 \qquad \ldots. (3.3.25)$$

$$\text{and } [D^4+2D^3+(\psi'-1)D^2+(\psi'-2)D+\lambda]\,Y'_{1l} = 0 \qquad \ldots. (3.3.26)$$

The solution of equations (3.3.25) and (3.3.26) are assumed to be of the form :

$$Y'_1 = ce^{\alpha v} \qquad \ldots. (3.3.27)$$

Then the characteristic equations are obtained as :

$$\alpha^4 + 2\alpha^3 + (\psi' - 1)\alpha^2 + (\psi' - 2)\alpha + r_o\lambda = 0 \qquad \ldots. (3.3.28)$$

and

$$\alpha^4 + 2\alpha^3 + (\psi' - 1)\alpha^2 + (\psi' - 2)\alpha + \lambda = 0 \qquad \ldots. (3.3.29)$$

Equations (3.3.28) and (3.3.29) are solved by Brown's method for solving Fourth order polynomial (McCormic and Salvodari, 1964) and the roots have been checked by Bairstow's method (Carnahan, Luther and Wilkes, 1969).

The general form the roots α_1 to α_4 is :

$$\alpha_1 \text{ to } \alpha_4 = -\tfrac{1}{2} \pm \sigma \pm i\tau \qquad \ldots. (3.3.30)$$

in which numerical values of σ and τ depend on the values of ψ' and λ . However in all cases the form of roots remains the same as equation (3.3.30).

The general solutions of equations (3.3.22) and (3.3.23) after resubstituting the values of v and η are (Ince 1956, Kamke 1959, Kosko 1965) :

$$Y'_{1u} = (1+d'Z)^{-\frac{1}{2}} \{ A \operatorname{Cosh}[\sigma \ln(1+d'Z)] \operatorname{Cos}[\tau \ln(1+d'Z)]$$

$$+ B \operatorname{Cosh}[\sigma \ln(1+d'Z)] \operatorname{Sin}[\tau \ln(1+d'Z)] + C \operatorname{Sinh}[\sigma \ln(1+d'Z)]$$

$$\operatorname{Cos}[\tau \ln(1+d'Z)] + D \operatorname{Sinh}[\sigma \ln(1+d'Z)] \operatorname{Sin}[\tau \ln(1+d'Z)]\}$$

$$\ldots. (3.3.31)$$

and

$$Y'_{11} = (1+d'Z)^{-\frac{1}{2}}\{A'\operatorname{Cosh}[\sigma'\ln(1+d'Z)]\operatorname{Cos}[\tau'\ln(1+d'Z)]$$

$$+ B'\operatorname{Cosh}[\sigma'\ln(1+d'Z)]\operatorname{Sin}[\tau'\ln(1+d'Z)] + C'\operatorname{Sinh}[\sigma'\ln(1+d'Z)]$$

$$\operatorname{Cos}[\tau'\ln(1+d'Z)] + D'\operatorname{Sinh}[\sigma'\ln(1+d'Z)]\operatorname{Sin}[\tau'\ln(1+d'Z)]\} \quad \ldots. (3.3.32)$$

The constants A, B, C, D, A', B', C' and D' are evaluated from the boundary conditions at the tip and top of the pile and from the compatibility conditions at interface between the two soil layers.

For the case (i) i.e. for freehead pile acted upon by shear force, Q_t, and axial load, P, the boundary conditions are :

I. at $x = 0$ i.e. at $Z = 0$

a) Bending moment = 0

$$EI(x)\frac{d^2y_u}{dx^2} = 0 \quad \ldots. (3.3.33a)$$

or

$$\frac{d^2Y'_{1u}}{dZ^2} = 0 \quad \ldots. (3.3.33)$$

and

b) S.F = Q_t

$$\frac{d}{dx}[-EI(x)\frac{d^2y_u}{dx^2}] - P(x)\frac{dy_u}{dx} = -Q_t \quad \ldots. (3.3.34\ a)$$

or

$$\frac{d^3Y'_{1u}}{dZ^3} + 4d'\frac{d^2Y'_{1u}}{dZ^2} + U\frac{dY'_{1u}}{dZ} = 1 \quad \ldots.(3.3.34)$$

II at $x = L_1$ or $Z = Z_1$. The 4 compatibility conditions are :

a) Deflection compatibility.

$$Y'_{1u} = Y'_{1l} \qquad \text{.... (3.3.35)}$$

b) Slope compatibility

$$\frac{dY'_{1u}}{dZ} = \frac{dY'_{1l}}{dZ} \qquad \text{.... (3.3.36)}$$

c) Bending moment compatibility.

$$\frac{d^2Y'_{1u}}{dZ^2} = \frac{d^2Y'_{1l}}{dZ^2} \qquad \text{.... (3.3.37)}$$

d) Shear force compatibility.

and $$\frac{d^3Y'_{1u}}{dZ^3} = \frac{d^3Y'_{1l}}{dZ^3} \qquad \text{.... (3.3.38)}$$

III. at $x = L$ or $Z = Z'$

a) B.M = 0

i.e. $$\frac{d^2Y'_{1l}}{dZ^2} = 0 \qquad \text{.... (3.3.39)}$$

and

b) S.F = 0

$$\frac{d^3Y'_{1l}}{dZ^3} + \frac{4d'}{(1+d'Z')}\frac{d^2Y'_{1l}}{dZ^2} + \frac{U}{(1+d'Z')^2}\frac{dY'_{1l}}{dZ} = 0 \qquad \text{.... (3.3.40)}$$

By using the above 8 boundary conditions, equations (3.3.33) to (3.3.40), the constants A, B, C, D, A', B', C' and D' are evaluated.

For the case (ii) i.e. free head pile acted upon by a moment, M_t and an axial load, P at the top and case (iii) i.e. fixed head pile acted up on by a lateral load, Q_t and axial load, P, the general form of the solutions, equations (3.3.31) and (3.3.32) remain same. The boundary conditions at x = L and compatibility conditions at $x = L_1$ also remain same. However the boundary conditions at x = 0 are :

Case (ii) For free head pile, M_t and P acting at top :

at x = 0

a) S.F. = 0 ....(3.3.41a)

$$\text{i.e.}\quad \frac{d^3Y'_{2u}}{dZ^3} + 4d'\frac{d^2Y'_{2u}}{dZ^2} + U\frac{dY'_{2u}}{dZ} = 0 \qquad \text{....(3.3.41)}$$

b) B.M = M_t .... (3.3.42a)

$$\text{i.e.}\quad \frac{d^2Y'_{2u}}{dZ^2} = 1 \qquad \text{.... (3.3.42)}$$

Case (iii) for fixed head pile, only Q_t and P acting at top :

at x = 0

a) slope = 0 .... (3.3.43a)

$$\text{i.e.}\quad \frac{dY'_{1f}}{dZ} = 0 \qquad \text{.... (3.3.43)}$$

b) $S.F = Q_t$ .... (3.3.44a)

i.e. $$\frac{d^3Y'_{1f}}{dZ^3} + 4d' \frac{d^2Y'_{1f}}{dZ^2} + U \frac{dY'_{1f}}{dZ} = 1 \qquad \text{.... (3.3.44)}$$

3.4. RESULTS AND DISCUSSION

Based on the generalized solutions presented above, exhaustive numerical results are obtained for the three different cases viz :

1) Free-Free pile with only P and Q_t acting at top
2) Free-Free pile with only P and M_t acting at top
and 3) Fixed-Free pile with P and Q_t acting at top.

Matlock and Reese (1960), Davisson (1960), Davisson and Gill (1963), Reddy and Valsangkar (1968) have shown that for all practical purposes, the laterally loaded pile with $Z' = 2$ or less behaves as a rigid member and with $Z' = 4$ or more behaves as a flexible member of infinite length for both free-free and fixed-free end conditions. Hence for obtaining the numerical results in this work, Z' is taken as equal to 4.0, which is a suitable approximation for most of the piles. Also, the value of c is taken as 0.6, which is the usual value for most of the field cases. For numerical computations the axial load coefficient U defined by equations (3.3.19) is expressed as a function of the critical load coefficient of a free-free pile of uniform cross-section without skin friction.

Results are presented for the value of $U = 0.3\ U_{cr}$ and $0.6\ U_{cr}$, where $U_{cr} = 0.9193$, which is the Buckling load coefficient for a pile of length $Z' = 4.0$. Thus in the analysis it is tacitly assumed that small deflection theory for beam-column is valid till this range. However, it should be noted that, since the solution obtained viz : Equations (3.3.31) and (3.3.32), are general in nature, any value of Z', c and U can be used for computation purposes. Exhaustive results are obtained for $r_o = 0.5, 1.0, 2.0, 4.0$ and 6.0 which represent a wide range of r_o found in the field. For each value of r_o results are obtained for different values of $Z_1 = 0.2$, 0.4 and 0.6.

The first set of results were obtained for $r_o = 1$ corresponding to uniform soil and the results were found to be in conformity with the results of Valsangkar (1969).

Figs. 3.9 and 3.10 show the effect of axial force on deflection and moment coefficient values for free-free pile for $Z_1 = 0.2$ and $r_o = 2.0$. It is seen from the Figs. 3.9 and 3.10 that there is a considerable increase in deflection and moment coefficient values with increase in axial force. Also as the axial load approaches the buckling load, the deflection and moment coefficient values get considerably magnified. Fig. 3.11 gives similar information for fixed free pile. For this case also for numerical computation U_{cr} is assumed to be equal to 0.9193. From these results it is seen that, as U/U_{cr} increases from 0.3 to 0.6, the increases in deflection and moment coefficient values for free-free pile are : 47.5 per cent and 69.6 per cent for maximum Y_1' and M_1' respectively, and 65.7 percent and 29.0 per cent for maximum Y_2' and M_2' respectively. The corresponding percent increases for fixed-head pile are 7 per cent and 5 percent for maximum Y_{1f}' and M_{1f}' respectively. Thus it is found that, under

the same axial loads the magnification in deflection and moment coefficient values in case of fixed-head pile is considerably lesser than that of free-head pile.

Figs. 3.12 to 3.17 represent the effect of r_o on deflection and moment coefficients, Y_1' and M_1', Y_2' and M_2' and M'_{1f} and Y'_{1f} for $Z_1 = 0.2$ and $\frac{U}{U_{cr}} = 0.3$ and 0.6. It is seen in general from these graphs that there is considerable increase in deflection and moment coefficient values for r_o less than unity and there is considerable dampening of deflection and moment coefficient values for r_o values greater than unity.

Figs. 3.18 and 3.19 depict the effect of varying r_o on deflection coefficients, Y_1' and Y_2', at $Z = 0$ for $Z_1=0.2$, 0.4 and 0.6, for $\frac{U}{U_{cr}} = 0.3$ and 0.6. Here again it is seen that for values of r_o less than unity there is considerable increase in deflection coefficients with increasing values of Z_1 and for values of r_o greater than unity there is considerable decrease in deflection coefficients with increasing values of Z_1. Also it is seen that for values of r_o greater than unity, there is relatively more decrease in deflection coefficients, Y_1' and Y_2' for Z_1 changing from 0.2 to 0.4 than when Z_1 changes from 0.4 to 0.6. From this it can be reasoned out that for r_o greater than unity as Z_1 approaches Z', the rate of decrease in deflection coefficients, Y_1' and Y_2' considerably decreases. This brings about the beneficial effect of having a hard layer at the top upto a certain depth, to control the deflection and moment coefficients.

Figs. 3.20 and 3.21 show the effect of r_o on maximum M_1' and M_2', for U/U_{cr} = 0.3 and 0.6, for different values of Z_1 = 0.2, 0.4 and 0.6. Here again for r_o values less than unity the maximum moment coefficients, M_1' and M_2' increase with increasing Z_1. For r_o values greater than unity upto 4.0, the maximum M_1' decreases with increasing Z_1, whereas for r_o values greater than 4.0 there is no exact correlation between r_o and Y_1' and Y_2' for different values of Z_1. This may be reasoned out from the fact that the maximum moment does not occur at a particular depth for all the values of Z_1. In Fig. 3.21 it is seen that, the effect of r_o on Max. M_2' is similar to that on M_1'.

Figs. 3.22 and 3.23 show the effect of increasing r_o on deflection and moment coefficients Y_{1f}' and M_{1f}' at Z = 0, for $\frac{U}{U_{cr}}$ = 0.3 and 0.6. Here also as expected for r_o values less than unity the deflection and moment coefficients increase with increasing Z_1 and for r_o values greater than unity, decrease with increasing Z_1.

3.5 CONCLUSIONS.

Based on the foregoing analysis and the numerical results presented, certain generalized conclusions can be drawn. The problem of tapered pile under generalized loading in layered soil presents the most general case of the pile problem met with; since the case has varying pile section properties, soil properties, and axial force. The analytical solutions obtained being in closed form can be used as a check on the convergence and numerical accuracies, obtained

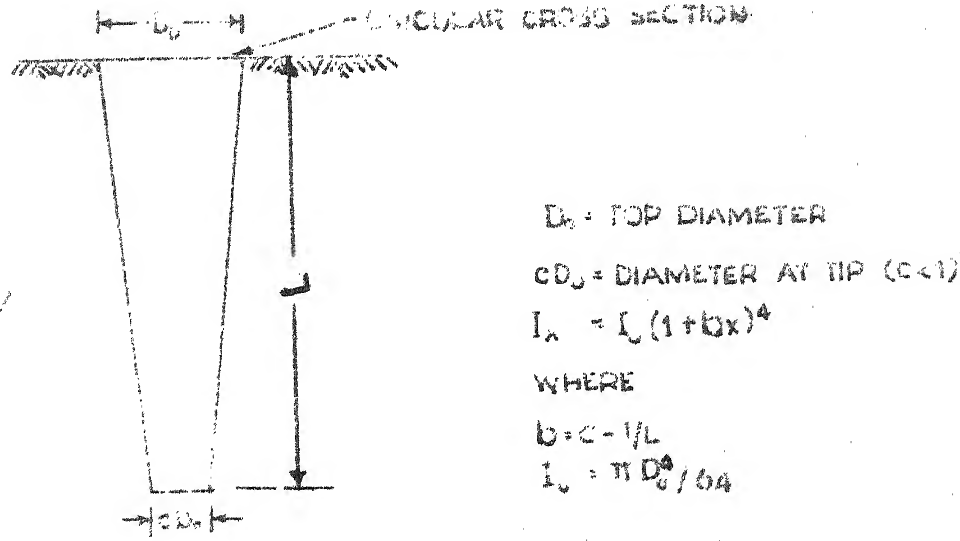


FIG. 3.1(a) UNIFORMLY TAPERED PILE WITH CIRCULAR CROSS - SECTION

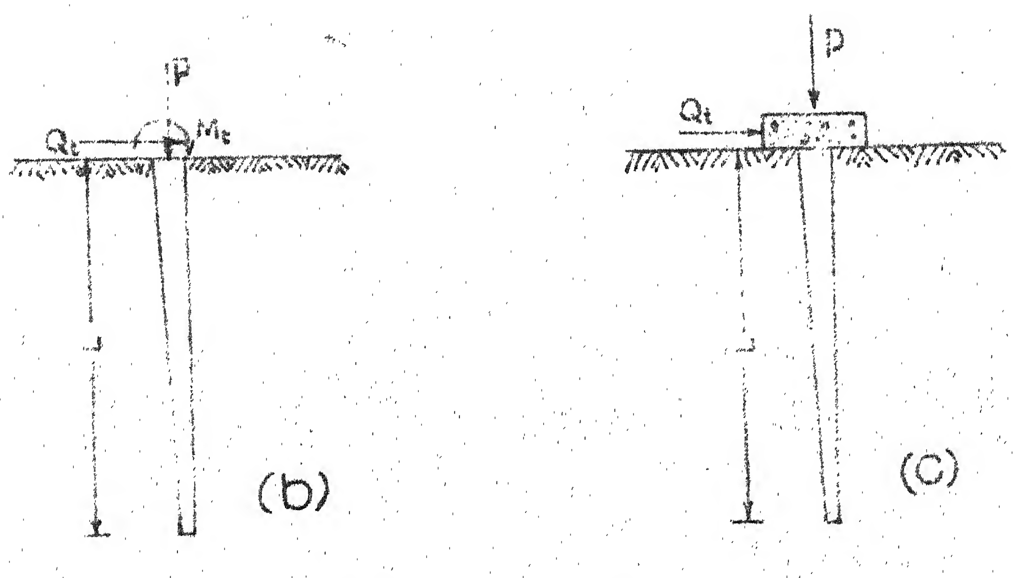


FIG. 3.1 FREE-FREE AND FIXED-FREE TAPERED PILES UNDER THE ACTION OF LATERAL AND AXIAL LOADINGS

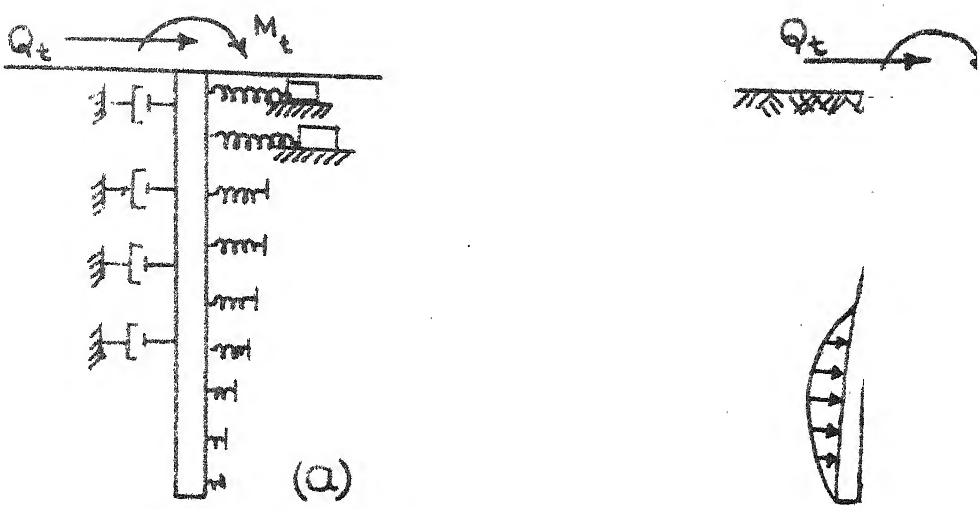


FIG. 3-2. REPRESENTATION OF GENERAL PROBLEM

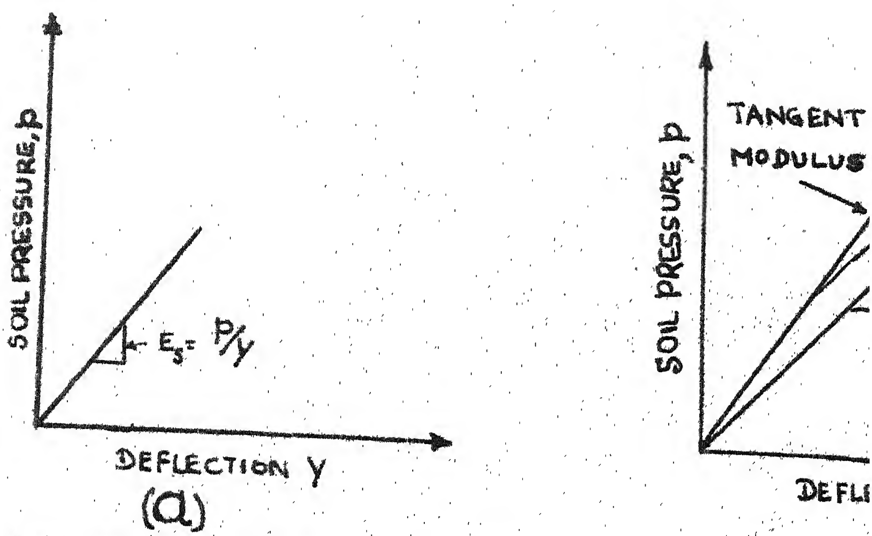


FIG. 3-3. TYPICAL LOAD - DEFLECTION

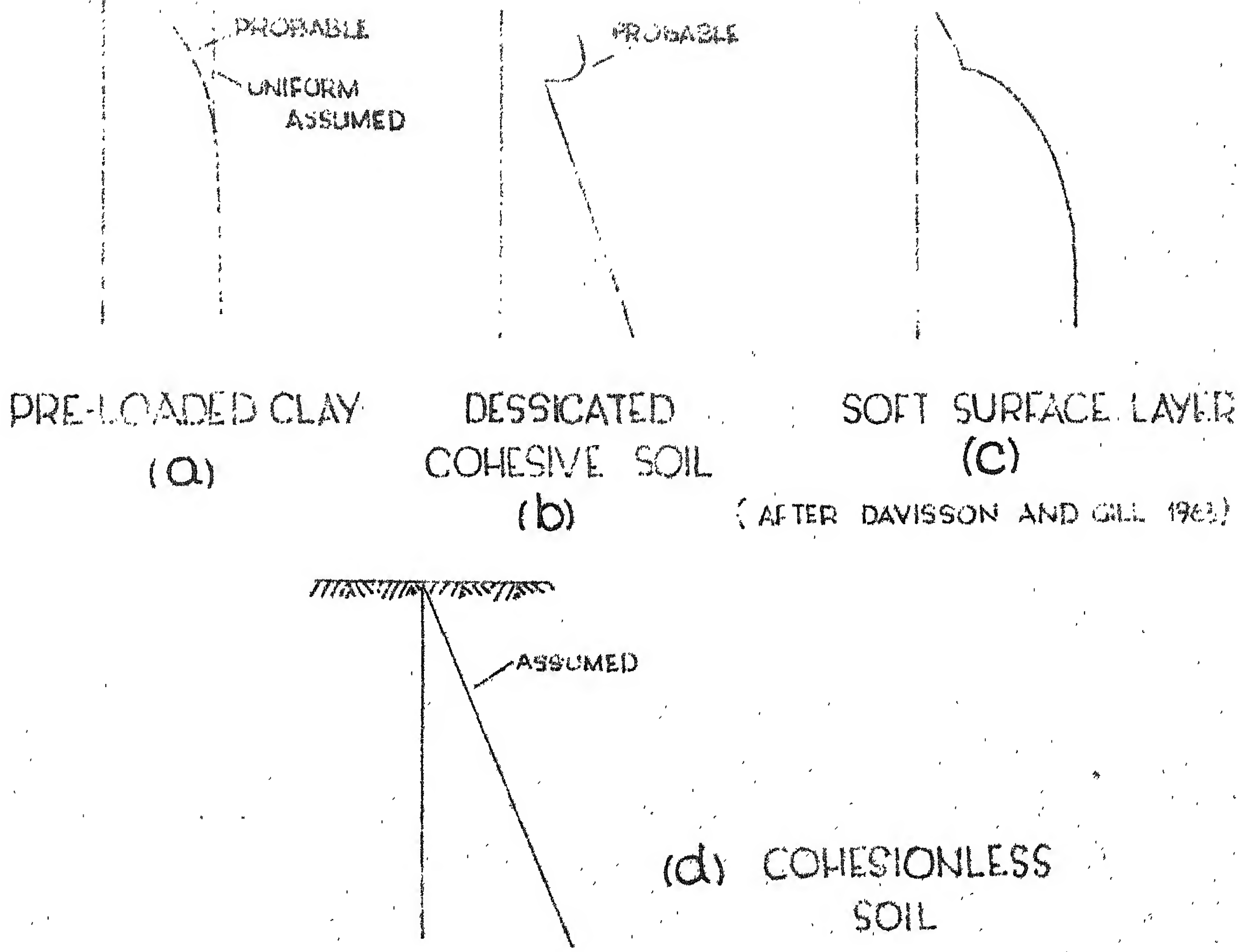
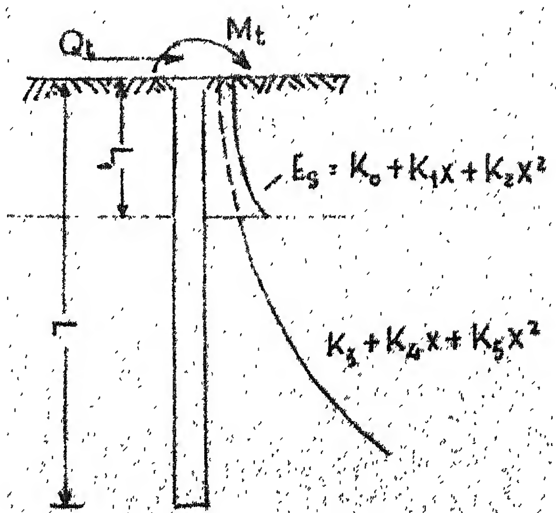


FIG. 3·4 VARIATION OF SOIL MODULUS WITH DEPTH.



.3·5 FREE HEAD PILE
TWO-LAYERED SOIL

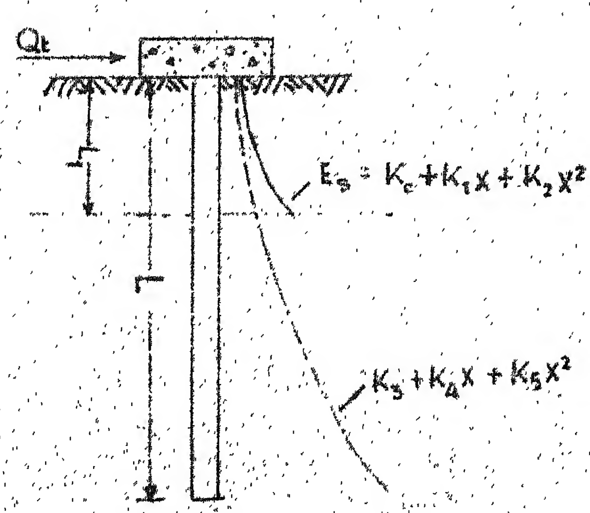


FIG. 3·6 FIXED HEAD PILE IN TWO LAYERED SOIL

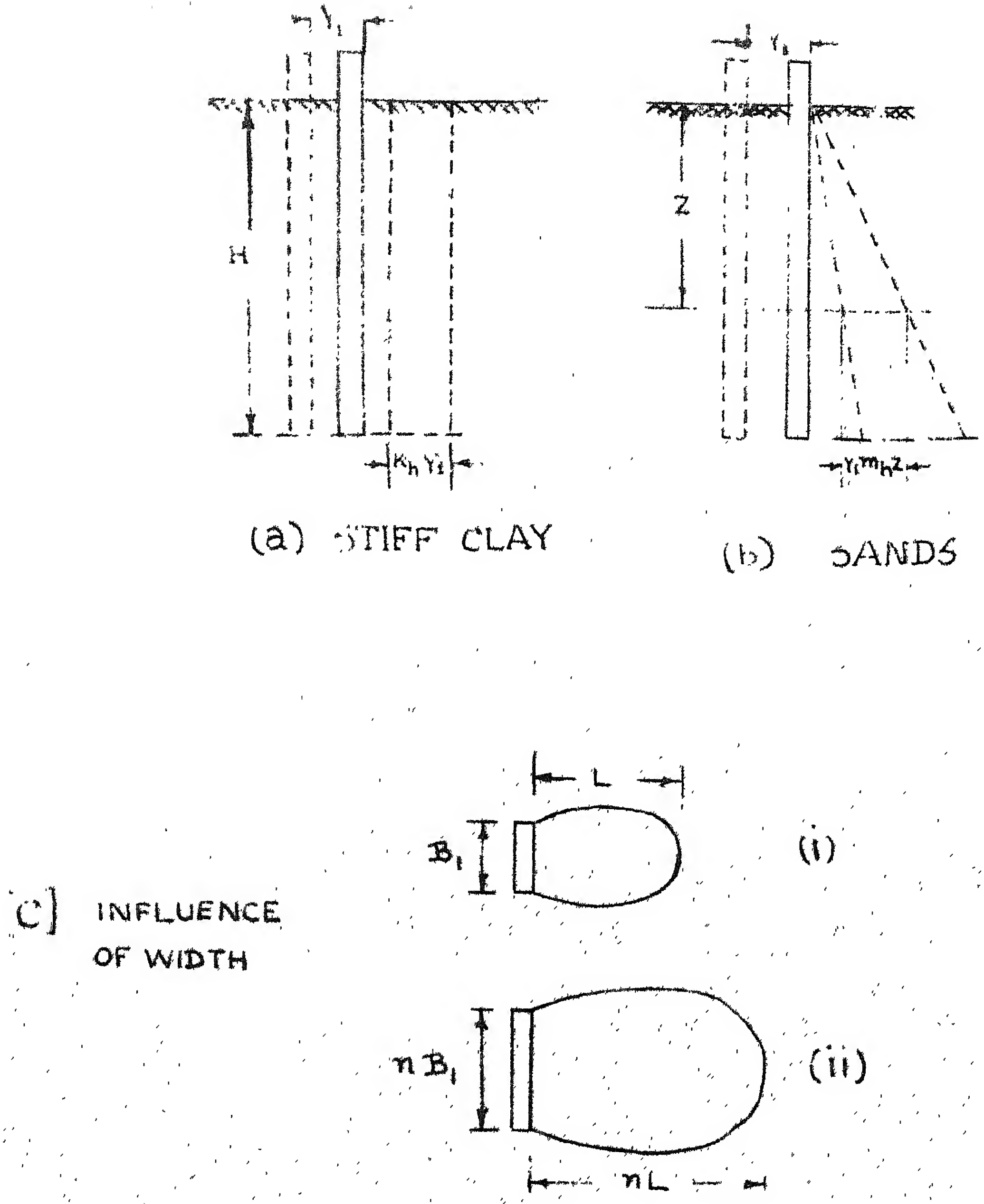


FIG 3·7 - VERTICAL BEAM EMBEDDED IN (a) STIFF CLAYS (b) SANDS & (C) INFLUENCE OF WIDTH OF BEAM ON DIMENSIONS OF BULB OF PRESSURE (AFTER TERZAGHI, 1955)

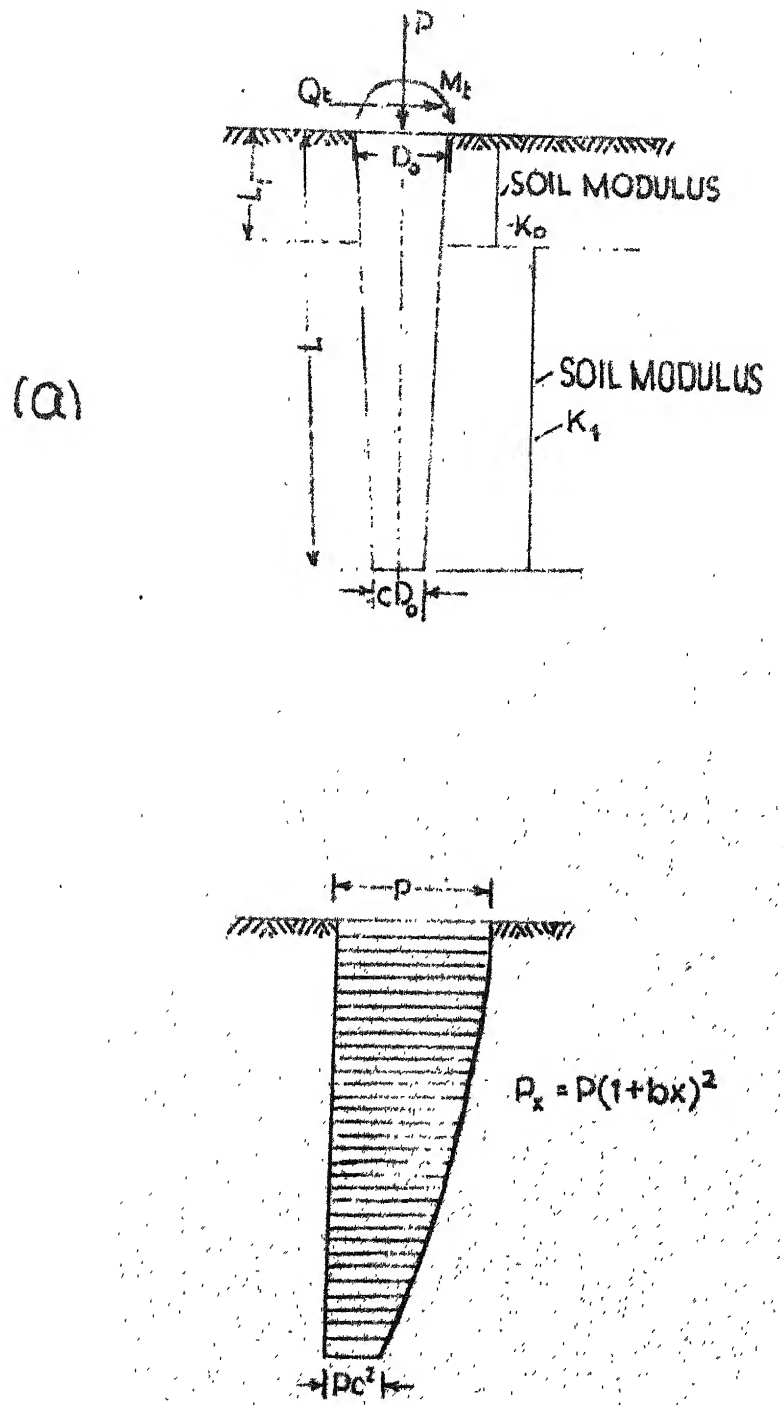


FIG.3-8 TAPERED PILE UNDER THE ACTION OF GENERALISED LOADING IN LAYERED SOIL AND VARIATION OF AXIAL FORCE.

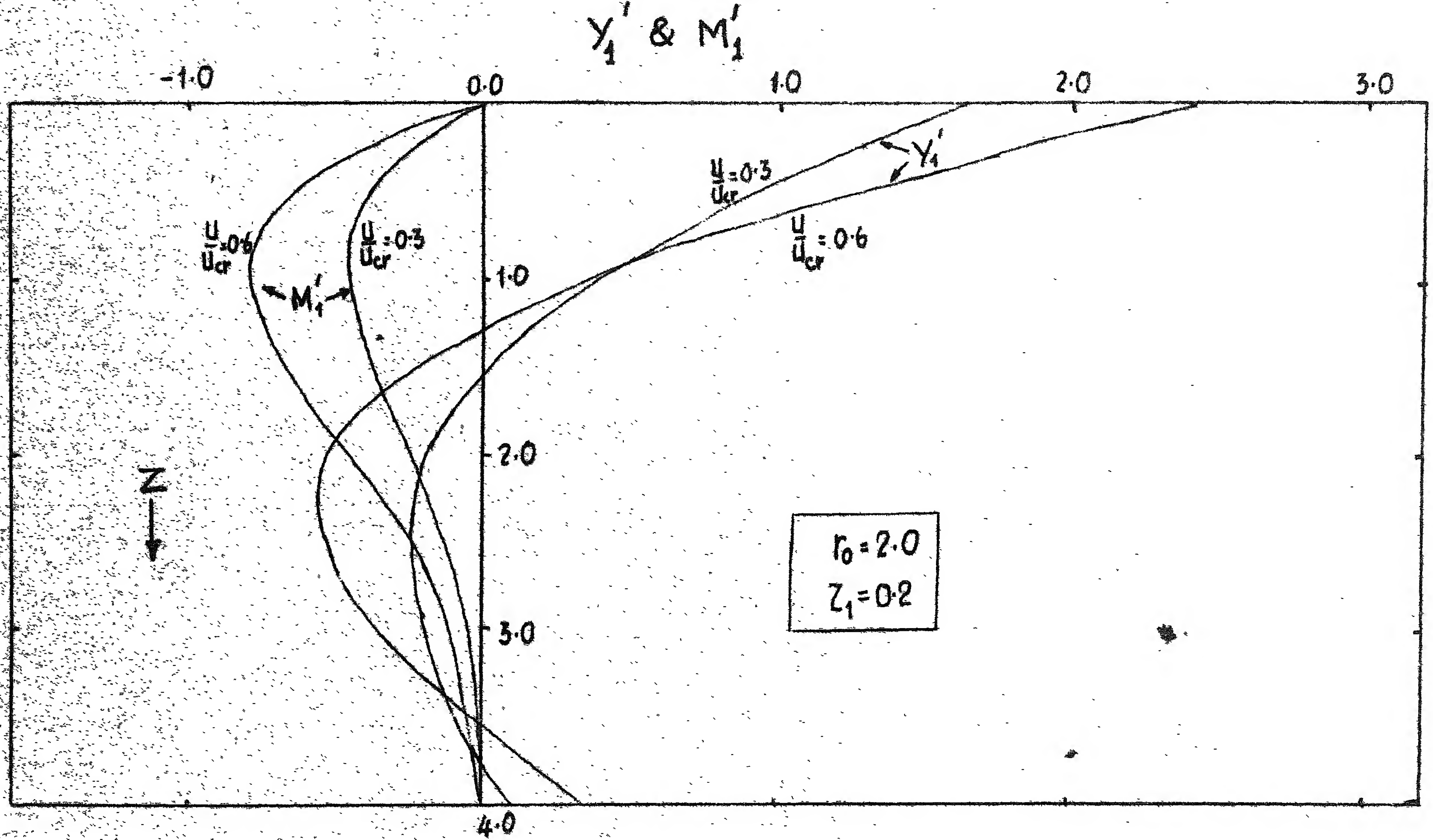


FIG. 3.9 EFFECT OF $\frac{u}{u_{cr}}$ ON Y_1' & M_1' FOR $r_0=2.0$ & $Z_1=0.2$

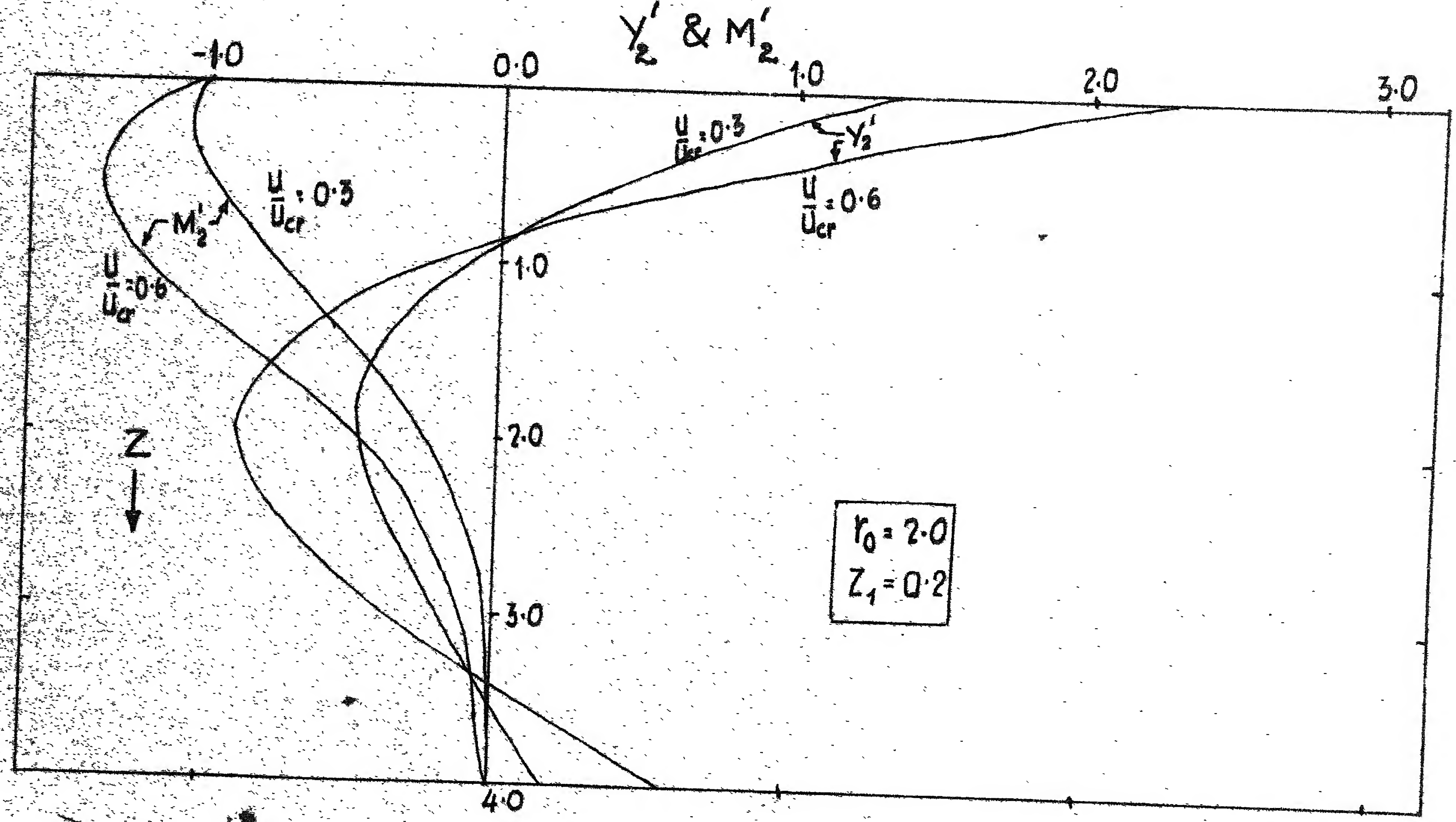


FIG.3.10 EFFECT OF $\frac{U}{U_{cr}}$ ON Y_2' & M_2' FOR $r_0 = 2.0$ & $Z_1 = 0.2$.

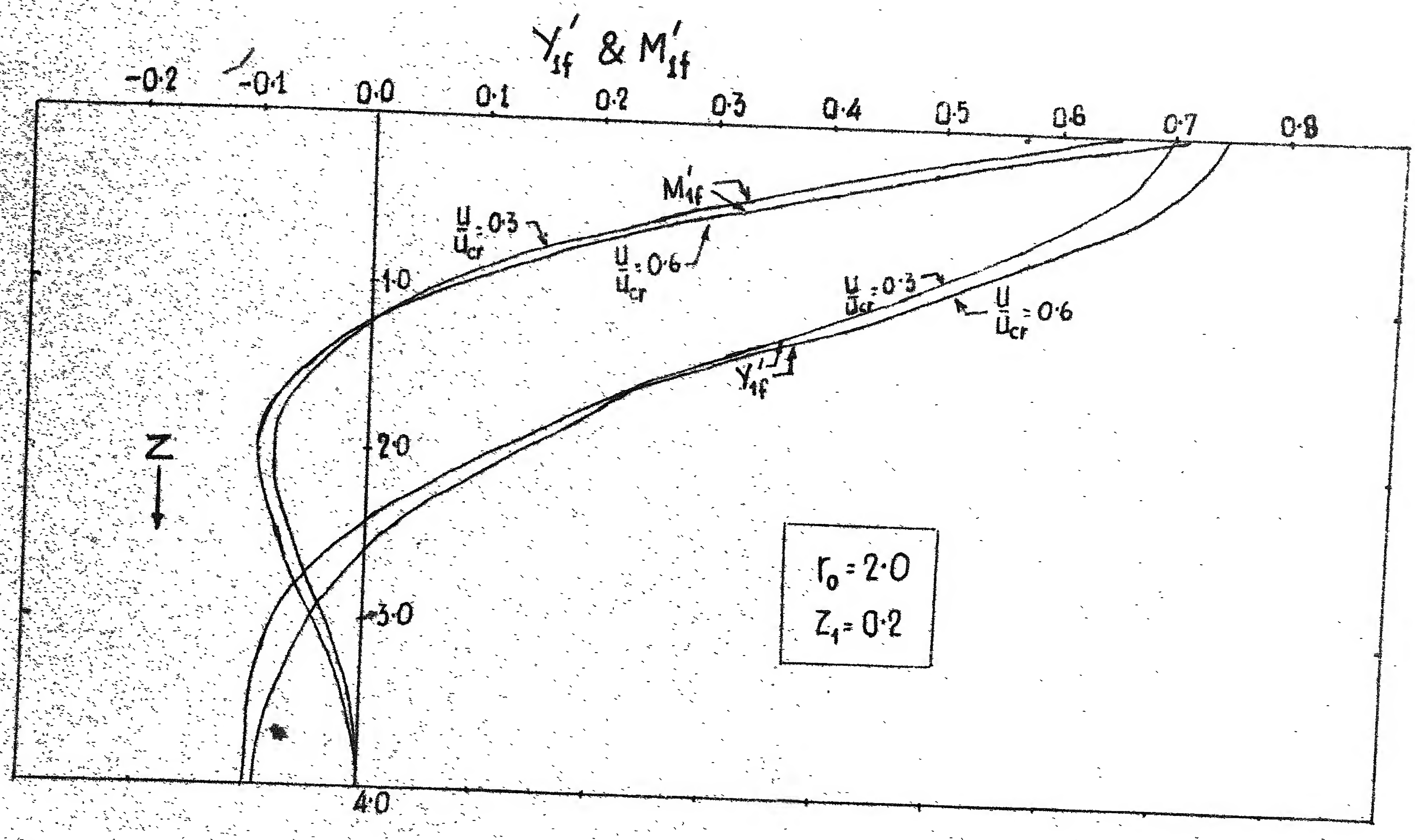


FIG. 3.11 EFFECT OF $\frac{U}{U_{cr}}$ ON Y'_{1f} & M'_{1f} FOR $r_0 = 2.0$ & $Z_1 = 0.2$.

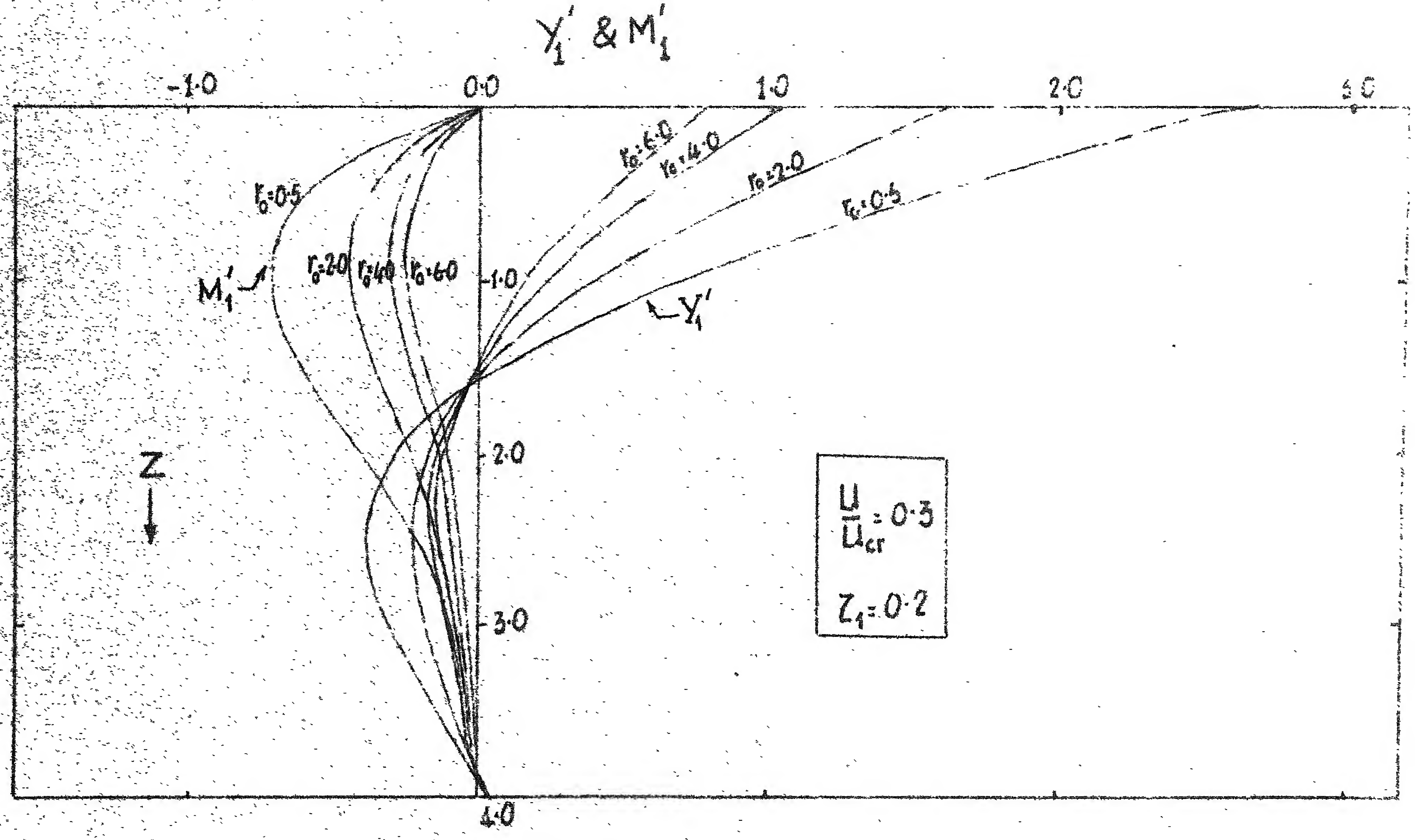


FIG.3.12 EFFECT OF r_0 ON Y_1' & M_1' FOR $\frac{U}{U_{cr}}=0.3$ & $Z_1=0.2$.

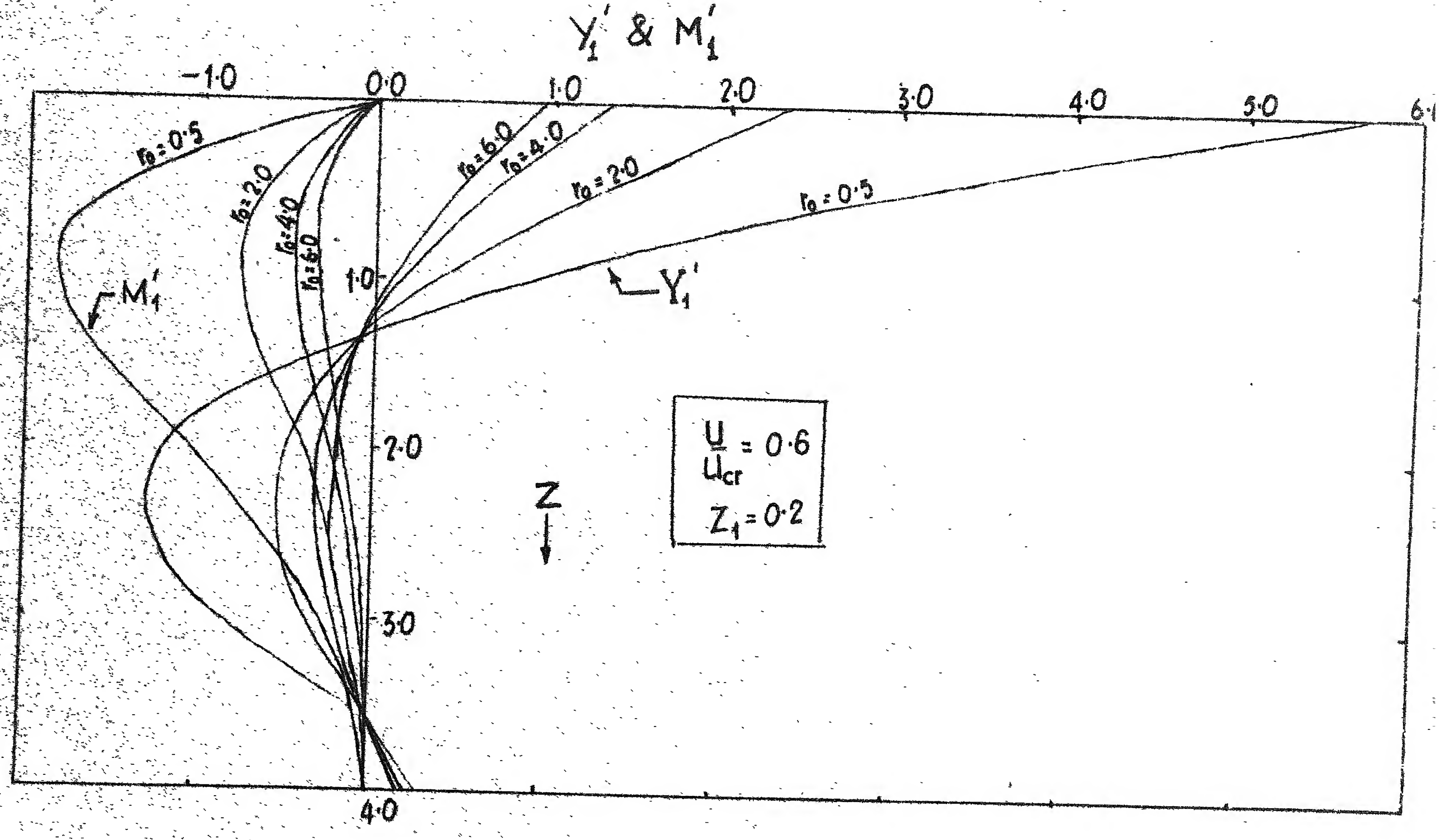


FIG. 3.13 EFFECT OF r_0 ON Y_1' & M_1' FOR $\frac{U}{U_{cr}} = 0.6$ & $Z_1 = 0.2$.

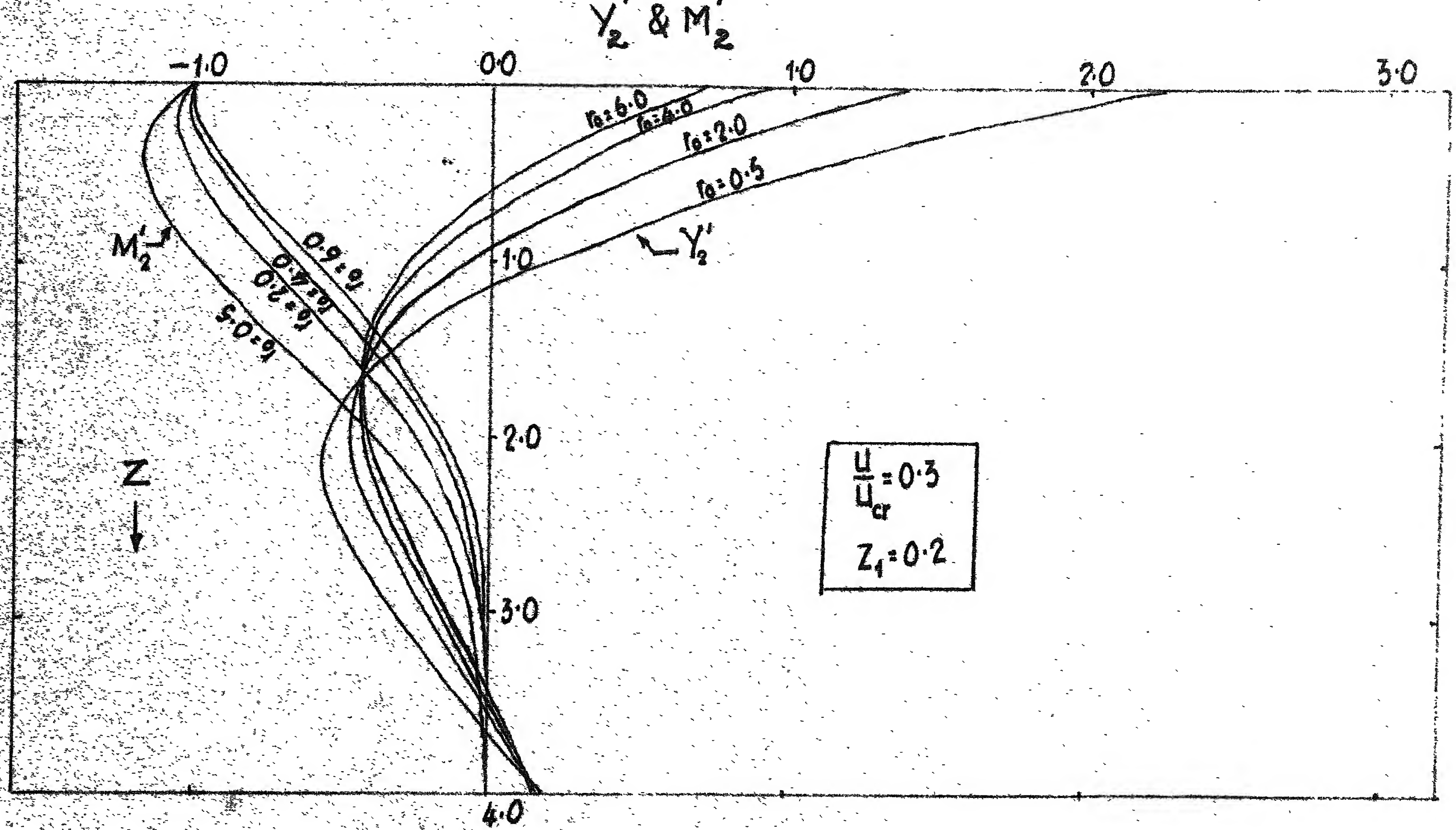


FIG.3.14 EFFECT OF r_0 ON Y_2' & M_2' FOR $\frac{U}{U_{cr}}=0.3$ & $Z_1=0.2$.

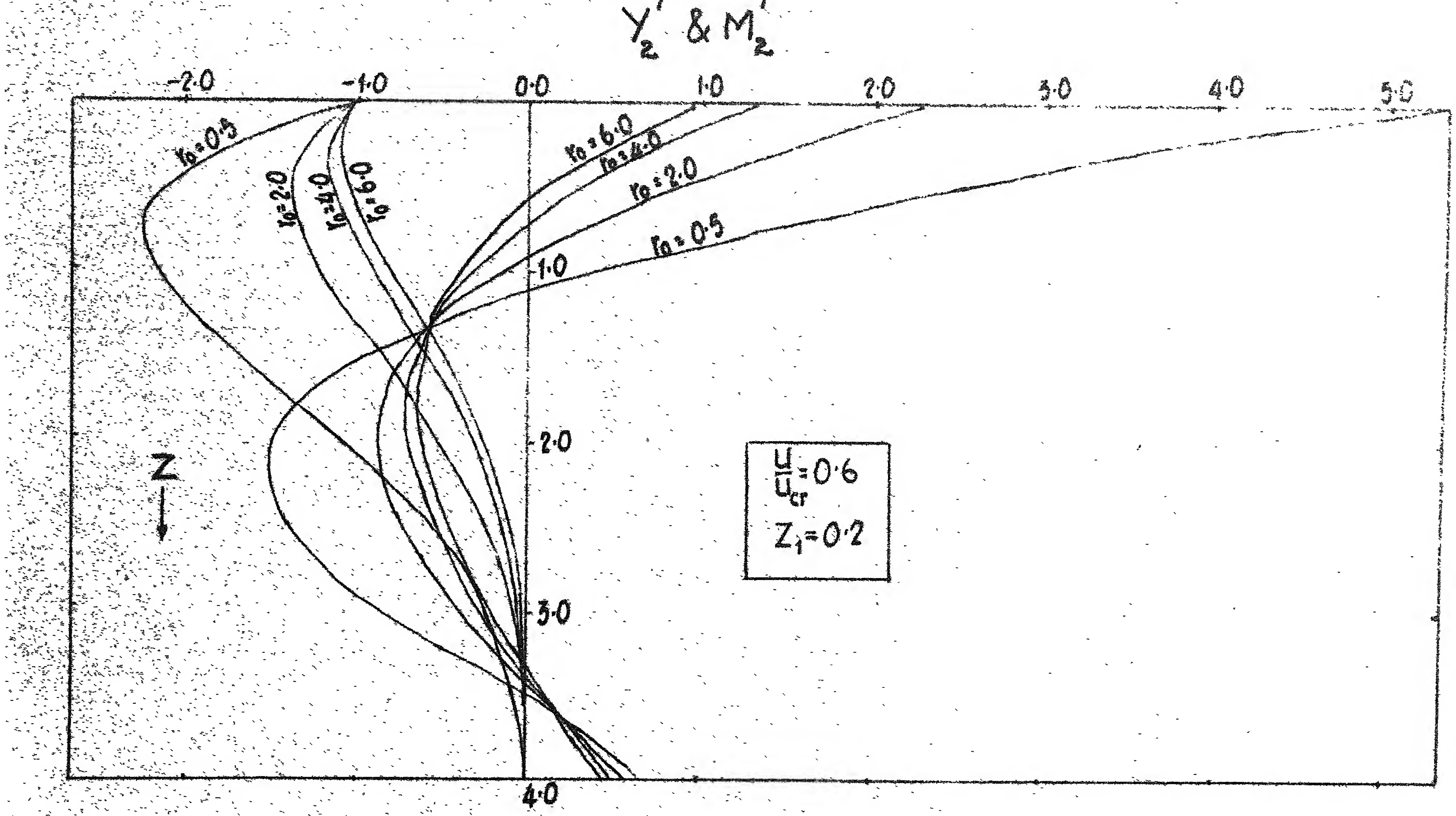


FIG. 3.15 EFFECT OF r_0 ON Y_2' & M_2' FOR $\frac{U}{U_{cr}} = 0.6$ & $Z_1 = 0.2$.

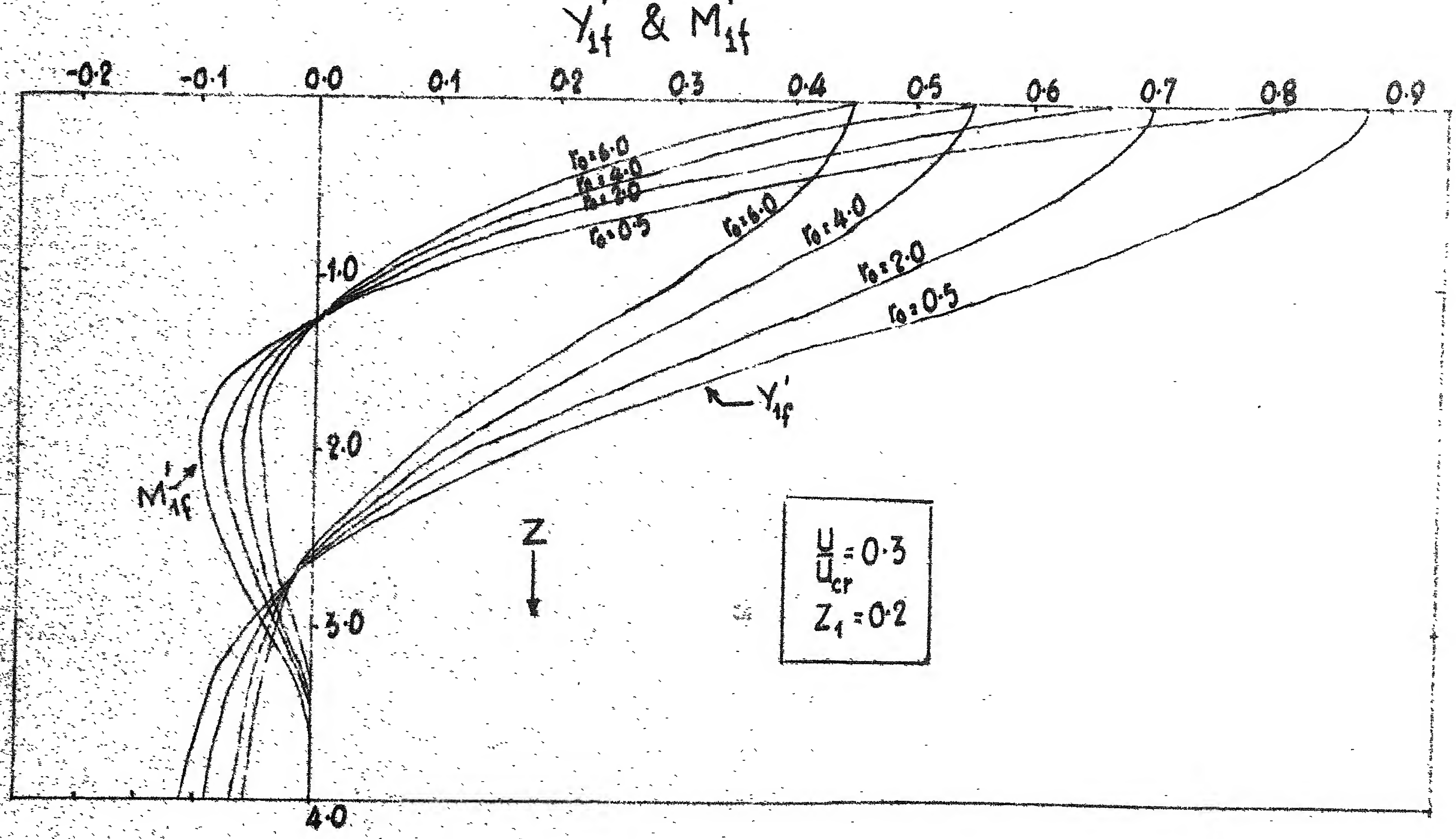


FIG.3.16 EFFECT OF r_0 ON Y'_{1f} & M'_{1f} FOR $\frac{U}{U_{cr}}=0.3$ & $Z_1=0.2$.

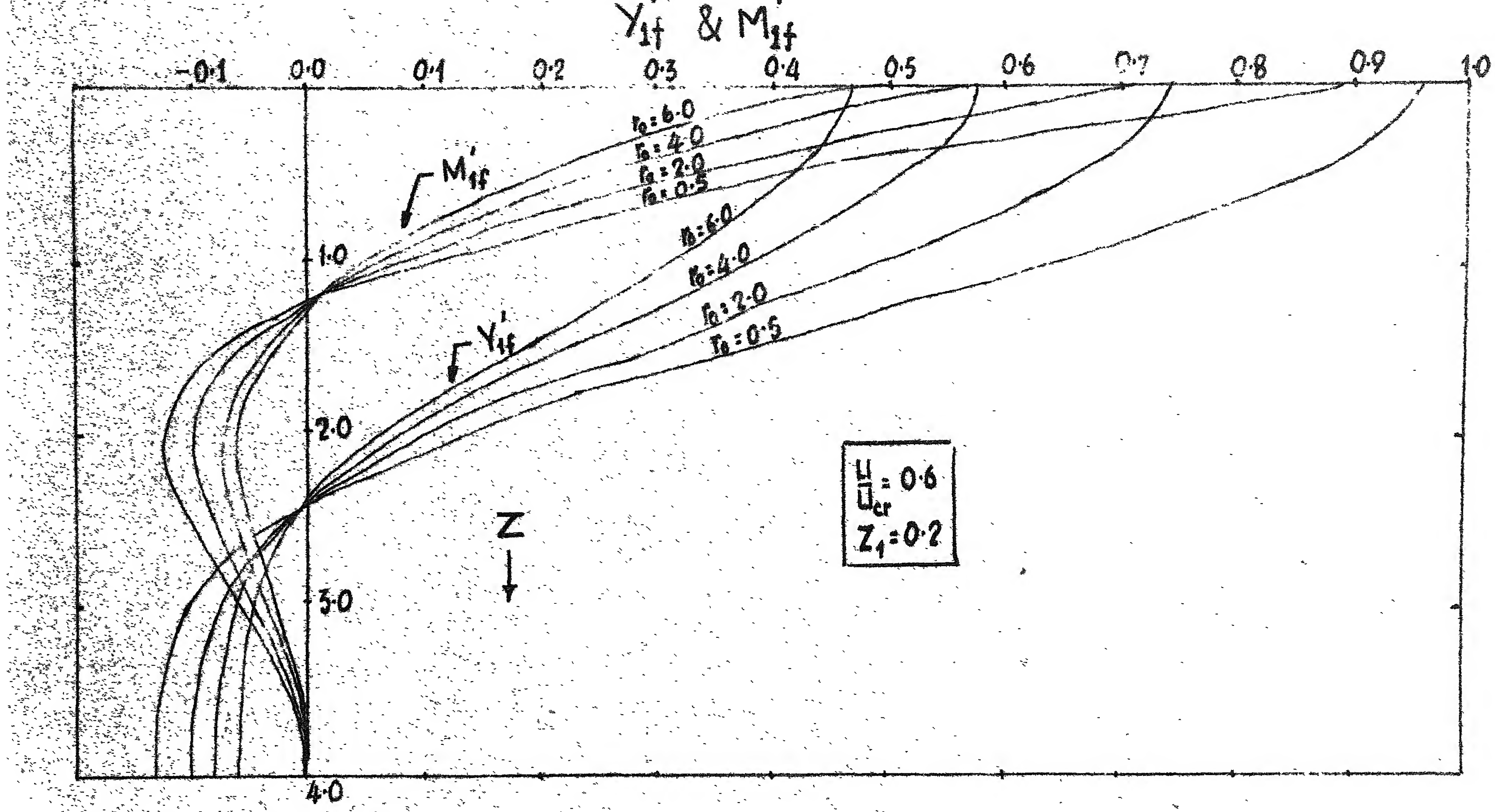


FIG.3.17 EFFECT OF r_0 ON Y'_{1f} & M'_{1f} FOR $\frac{U}{U_{cr}} = 0.3$ & $Z_1 = 0.2$.

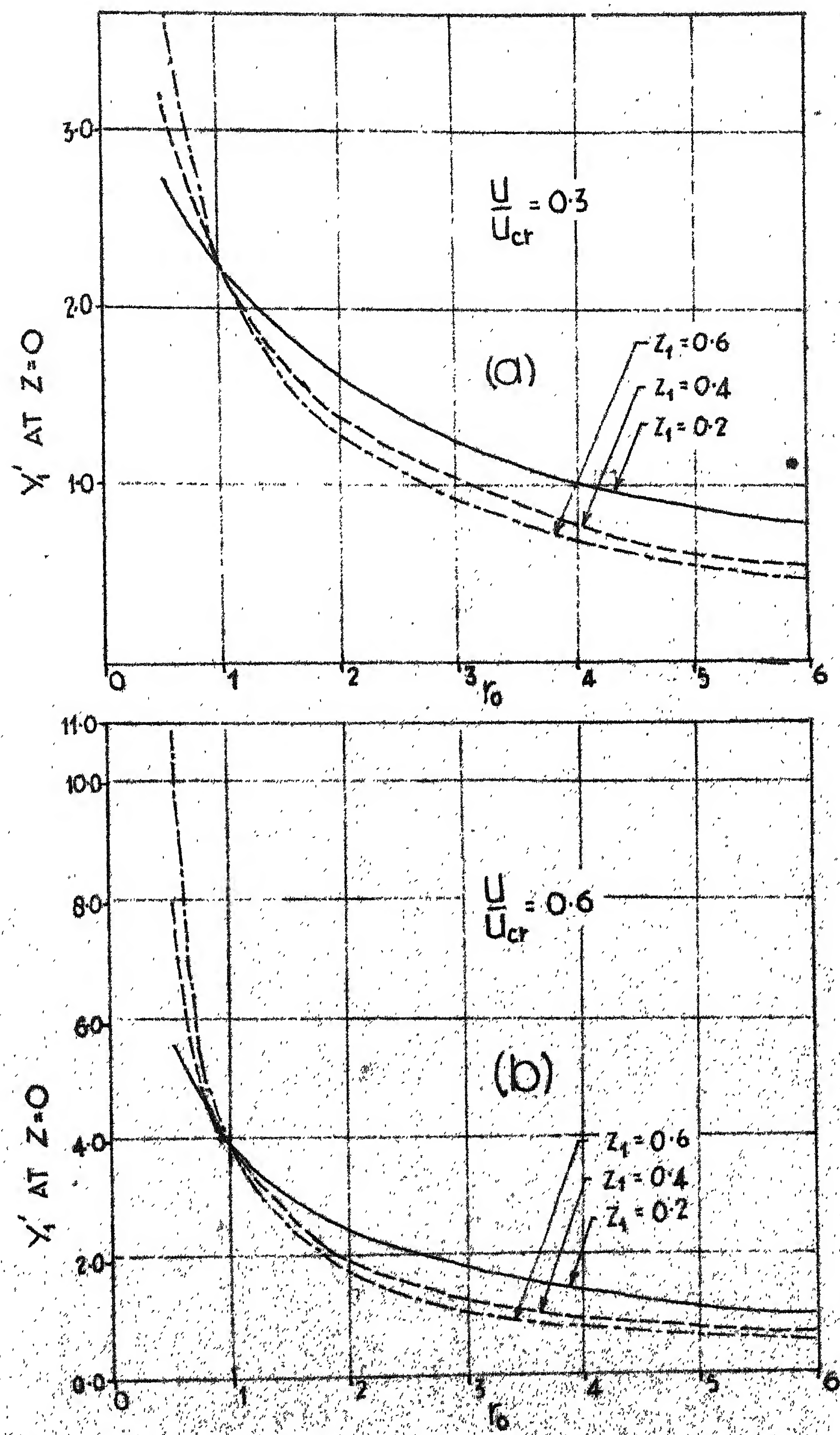


FIG. 3.18 EFFECT OF r_0 ON Y_1' AT $Z=0$

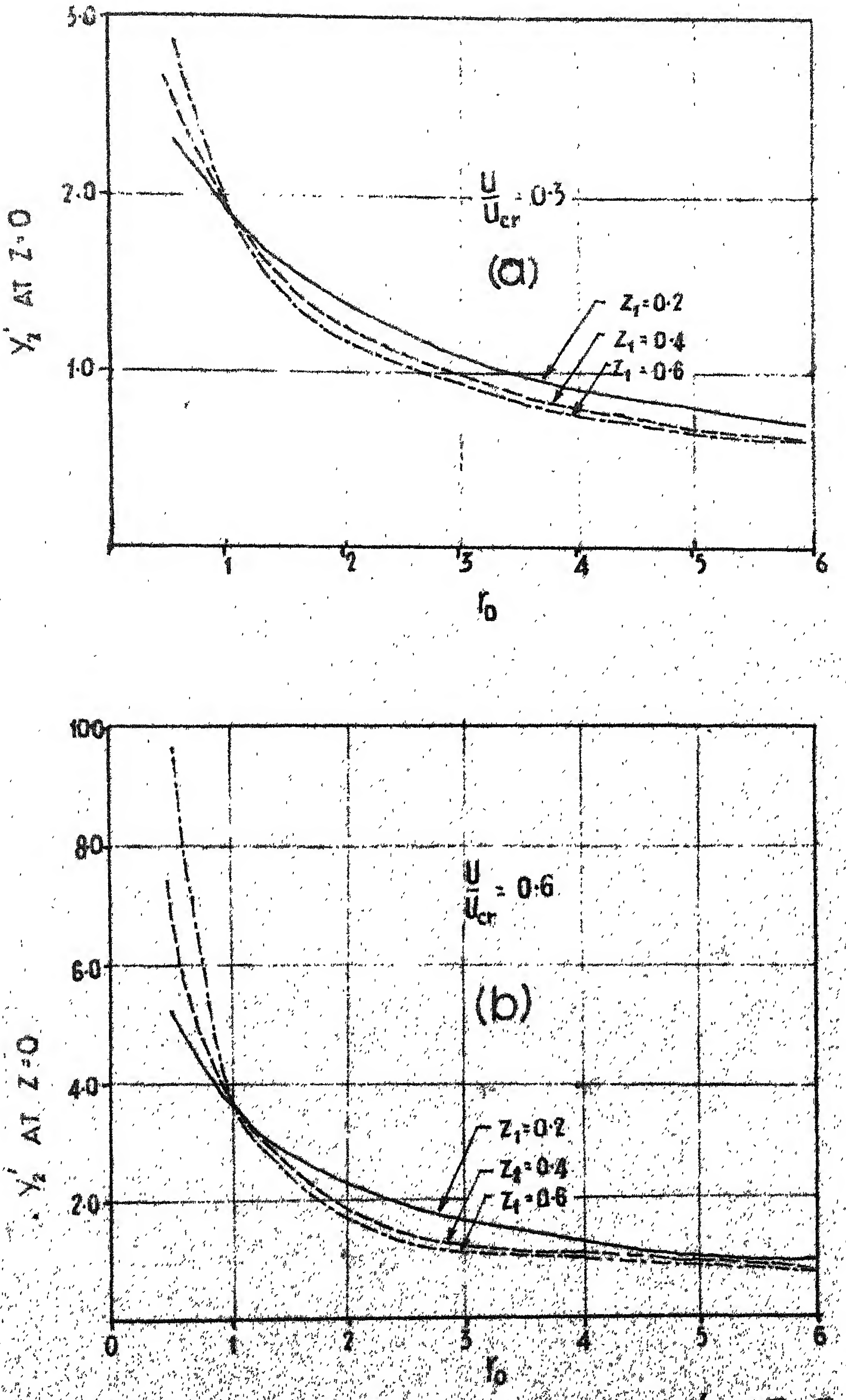
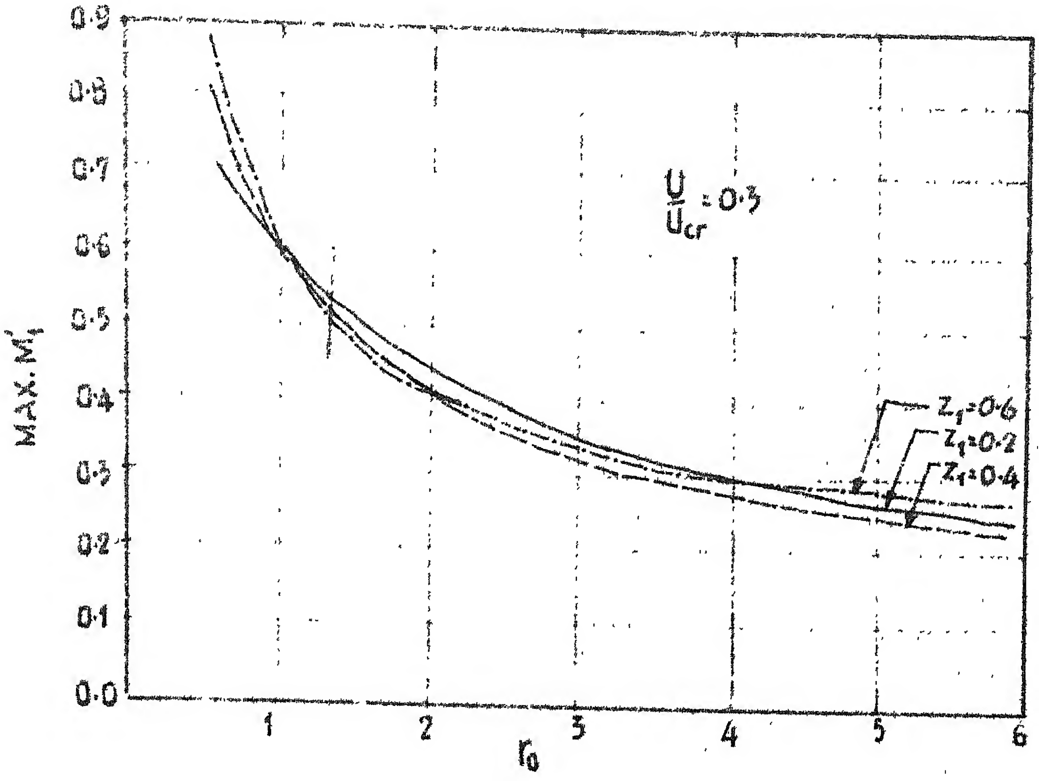


FIG 3.19 EFFECT OF r_0 ON Y_2' AT Z=O



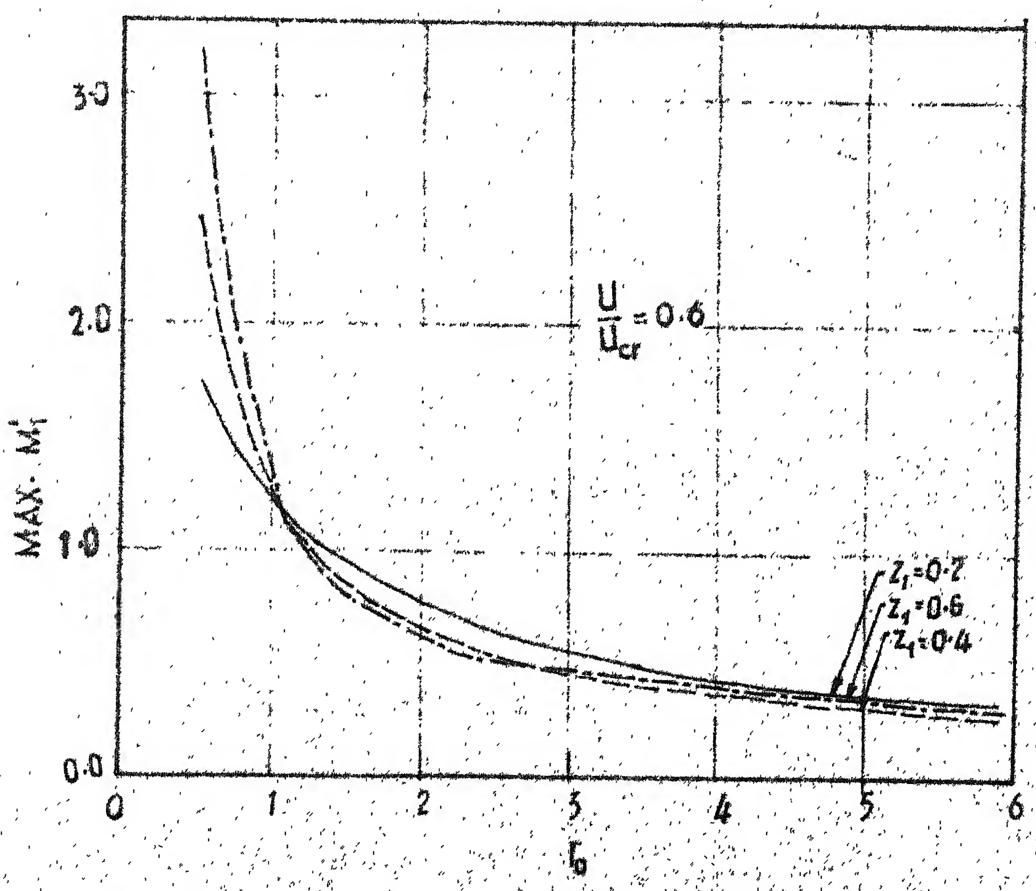


FIG 3.20 EFFECT OF r_0 ON MAX. M_1'

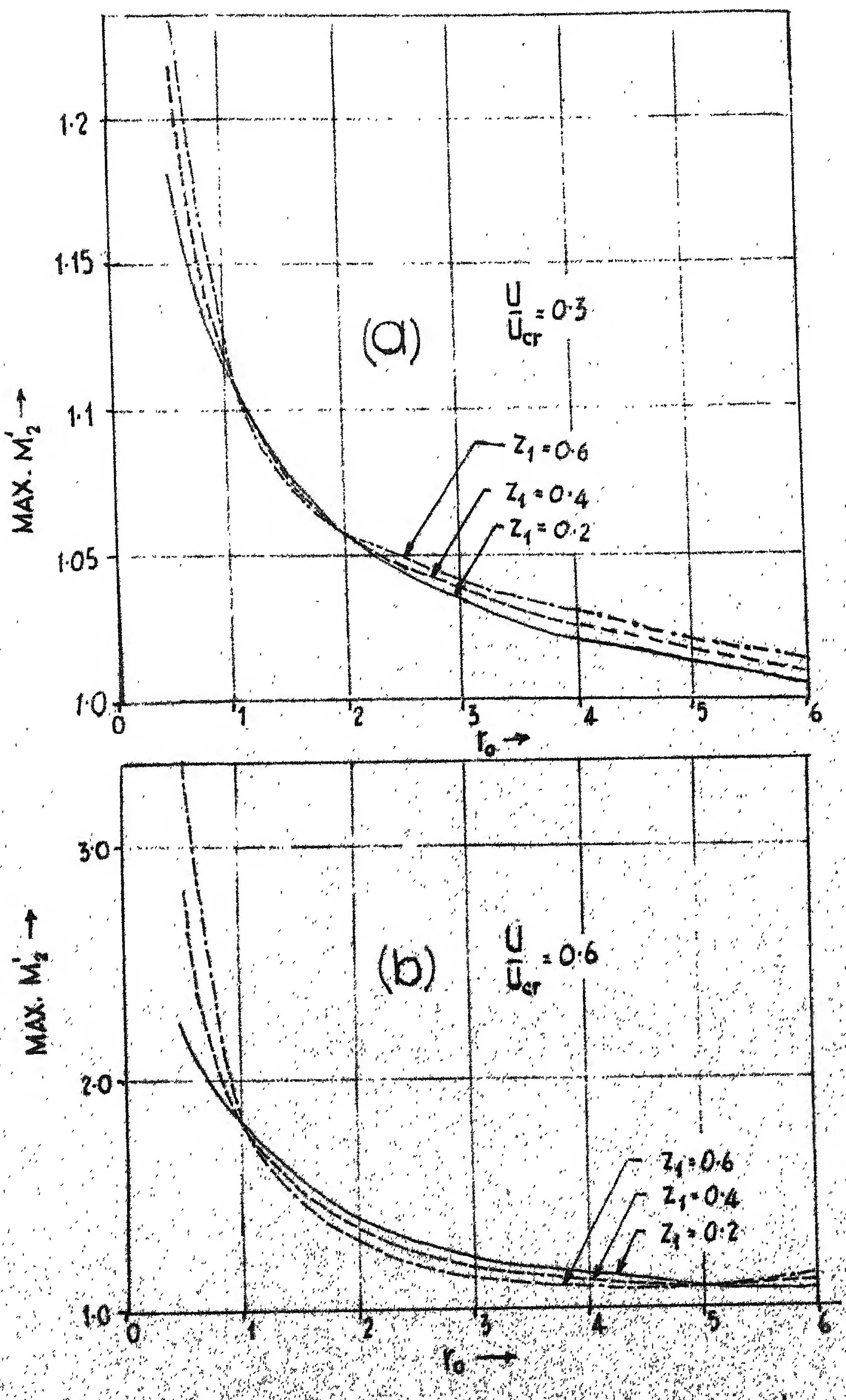


FIG. 3.21 EFFECT OF r_o ON MAX. M_2'

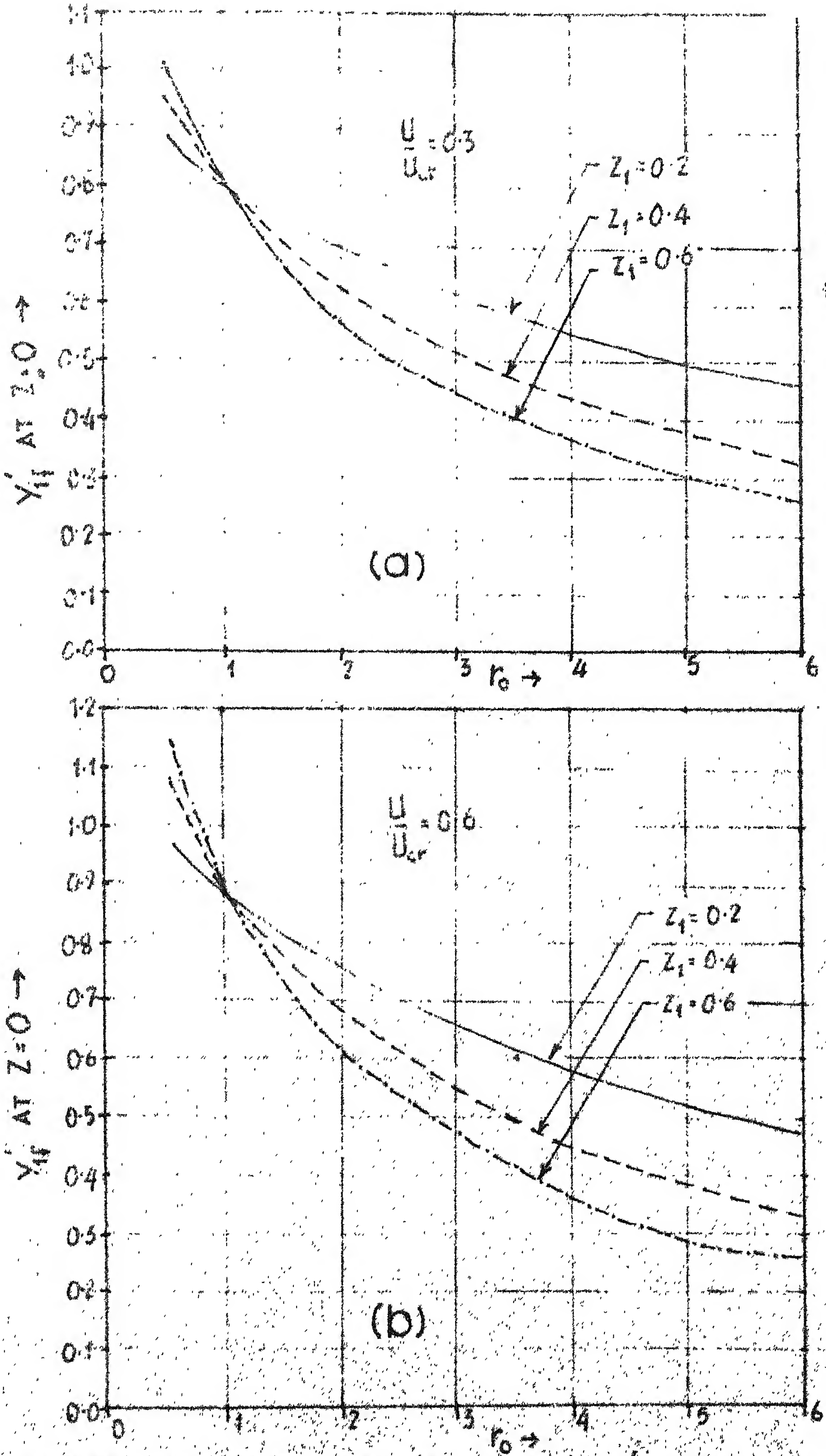
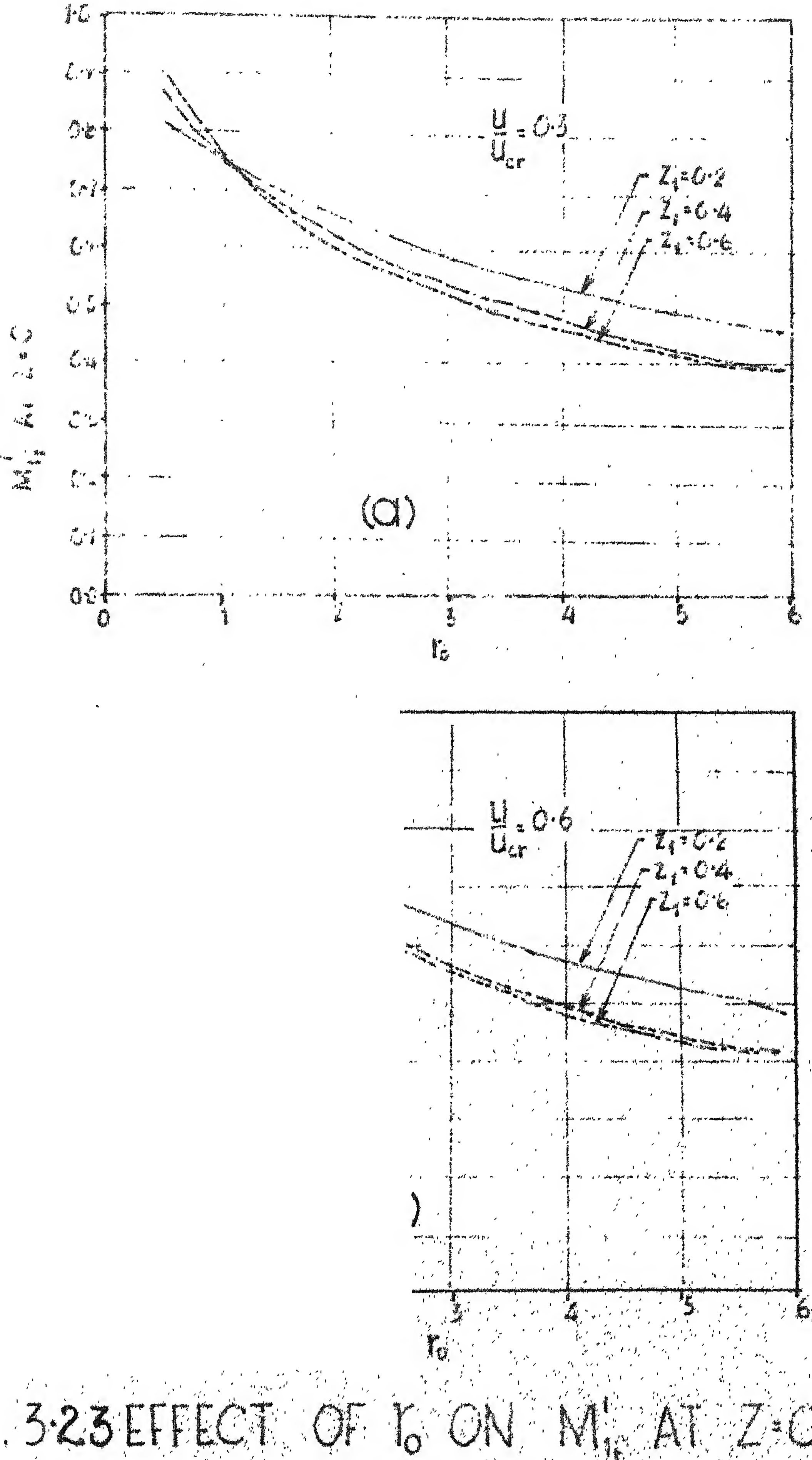


FIG. 3.22 EFFECT OF r_o ON Y'_{1f} AT $Z=O$



i. 3.23 EFFECT OF r_o ON M'_{1r} AT Z=O

CHAPTER 4

GENERALIZED SOLUTION FOR AXIALLY AND LATERALLY LOADED TAPPERED PILES IN ELASTO-PLASTIC SOIL

4.1 GENERAL

In Chapter 3 the studies were based on elastic theory which is valid at working loads, wherein p-y relationship was assumed to be linear according to Broms (1964 a ,1964 b). However, at higher loads the soil behaves non-linearly as reported by : Wagner 1953, Fitzgibbon 1956, Matlock and Ripperger 1957 and Mori 1964. To account for this in the present chapter the soil is assumed to be represented by linear combination of St. Venant's body and linear spring as shown in Fig.3.2. Fig. 4.2 represents the p-y relationship for this model. Mori (1964) has presented the solutions for a laterally loaded pile problem with the above assumption for infinitely long pile and the correlation with field data was seem to be reasonably good. Earlier Starzewski (1959) observed from the experiments conducted on beams loaded at edge and resting on soil that, the measured soil pressures were in between the pressure predicted by soil modulus concept and that obtained from assuming the p-y relationship as shown in Fig. 4.2.

Eventhough the fact that the soil at the top region of the pile goes into plastic region was recognized as early as 1953

(Wagner, 1953) , only recently Broms (1964a, 1964b) presented the solutions for the ultimate carrying capacity of laterally loaded piles taking into account the plastic resistance of the soil. In the analysis it was assumed that either pile fails because of excessive bending moments or the soil goes into plastic resistance along the embedded length of the pile. However in most of the cases usually only a few feet of the soil in the top goes into plastic resistance, which may not be sufficient for the pile failure to take place.

Reddy and Valsangkar (1970) presented the generalized solutions for laterally loaded pile in elasto-plastic soil. M.B. Roy (1970) presented studies on laterally loaded pile in elasto-plastic soil for constant soil modulus, and constant plastic resistance. Basudhar (1971) studied the effect of axial load also in addition to lateral load on the pile in elasto-plastic soil. Recently Domenico Lalli (1971) presented the solution for axially and laterally loaded pile of variable cross-section by large deflection matrix method wherein bi-linear p-y relationship is assumed. The results are presented in dimensional form for a pile of particular cross-section.

As, except for Lalli's (1971) work all other studies confine to uniform cross-section piles. In this study generalized solutions are presented for axially and laterally loaded tapered pile in elasto-plastic soil. The assumed variation of axial force along the length of the pile is shown in Fig. 4.1(b). This particular variation has been taken in the the analysis for the reasons already discussed in Chapter 3.

4.2 STATEMENT OF THE PROBLEM

Fig. 4.1 shows a circular uniformly tapered pile whose moment of inertia variation along the embedded length of the pile is :

$$I(x) = I_o(1+bx)^4 \qquad \text{.... (4.2.1)}$$

in which

$$I_o = \frac{\Pi D_o^4}{64} ;$$

$$b = \frac{c-1}{L} ;$$

c = ratio of the bottom diameter to the top diameter;

and D_o = diameter of the pile at the top.

In this schematic diagram the pile is acted upon at the top by (i) a lateral load Q_t and an axial load P.

(ii) a moment M_t and an axial load P.

The above two cases have been considered separately in the present chapter.

The variation of axial force shown in Fig. 4.1 (b) is given by:

$$P(x) = P(1+bx)^2 \qquad \text{.... (4.2.2)}$$

where P = axial force at the top of the pile.

Because of relatively weak nature of the soil and large deflections at the top, upto a certain depth the soil goes into plastic region as is concluded at the end of Chapter 2. To account for this

plastic resistance offered by the soil the load deflection relationship is assumed as shown in Fig. 4.2 in this analysis. Broms (1964 b) presented an analysis for the ultimate load carrying capacity of a laterally loaded pile in cohesionless soil with the variation of plastic resistance in the plastic zone with depth as

$$P_{ult} = 3\, B'\, \gamma x\, K_p \qquad \ldots. \ (4.2.3)$$

Reddy and Valsangkar (1970) presented a solution for ultimate soil resistance for a laterally loaded pile embedded in cohesionless soil with the assumption that, a triangular wedge moves upward at the time of failure as in the case of cohesive soil, the corresponding variation of ultimate resistance is parabolic in nature. Basudhar (1971) on the basis of the same assumptions as above presented generalized solution for axially and laterally loaded piles in elasto-plastic soil.

4.3 ANALYSIS.

The governing differential equations for the uniformly tapered pile shown in Fig. 4.1, wherein it is assumed that, the bottom layer has constant soil modulus and the top layer goes into plastic region, are given by :

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2 y_u}{dx^2}\right] + \frac{d}{dx}\left[P(x)\frac{dy_u}{dx}\right] + t = 0 \cdot \quad 0 \leq x \leq L_1 \qquad \ldots. \ (4.3.1)$$

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2 y_1}{dx^2}\right] + \frac{d}{dx}\left[P(x)\frac{dy_1}{dx}\right] + K_o y = 0. \quad L_1 \leq x \leq L \qquad \ldots. \ (4.3.2)$$

in which t = coefficient of ultimate soil resistance.

Substituting for $I(x)$ and $P(x)$ from equations 4.2.1 and 4.2.2 into equations 4.3.1 and 4.3.2, leads to :

$$\frac{d^2}{dx^2}\left[EI_o(1+bx)^4 \frac{d^2y_u}{dx^2}\right] + \frac{d}{dx}\left[P(1+bx)^2 \frac{dy_u}{dx}\right] + t = 0 \qquad .... (4.3.3)$$

$$0 \leq x \leq L_1$$

$$\frac{d^2}{dx^2}\left[EI_o(1+bx)^4 \frac{d^2y_1}{dx^2}\right] + \frac{d}{dx}\left[P(1+bx)^2 \frac{dy_1}{dx}\right] + K_o y_1 = 0 \qquad .... (4.3.4)$$

$$L_1 \leq x \leq L$$

Again to facilitate the numerical computations equations 4.3.3 and 4.3.4 are transformed to non-dimensional form by making use of the following non-dimensional parameters:

Depth coefficient; $Z = x/R$ .... (4.3.5)

$Z_1 = L_1/R$ .... (4.3.6)

and Minimum depth coefficient $Z' = L/R$ ..... (4.3.7)

in which R = relative stiffness factor given by

$$R = \sqrt[4]{EI_o/K_o} \qquad .... (4.3.8)$$

For the cases of free-free pile and fixed-free pile, the non-dimensional deflection and moment coefficients are defined as follows:

$$y_1' = \frac{y_1 EI_o}{tR^4} \qquad .... (4.3.9)$$

$$M_1' = \frac{M}{tR^2} \qquad \ldots. (4.3.10)$$

in which

y_1 = dimensional deflection ;

and M = dimensional bending moment.

By making use of the above non-dimensional coefficients equations 4.3.3 and 4.3.4 reduce to :

$$\frac{d^2}{dZ^2}[(1+d'Z)^4 \frac{d^2y'_{1u}}{dZ^2}] + U \frac{d}{dZ} [(1+d'Z)^2 \frac{dy'_{1u}}{dZ}] + 1 = 0$$

$$0 \leq Z \leq Z_1 \qquad \ldots. (4.3.11)$$

$$\frac{d^2}{dZ^2} [(1+d'Z)^4 \frac{d^2y'_{11}}{dZ^2}] + U \frac{d}{dZ} [(1+d'Z)^2 \frac{dy'_{11}}{dZ}] + y'_{11} = 0$$

$$Z_1 \leq Z \leq Z' \qquad \ldots. (4.3.12)$$

in which

$$d' = \frac{c-1}{(L/R)}$$

$$U = \frac{PR^2}{EI_o} \qquad \ldots. (4.3.13)$$

The above equations 4.3.11 and 4.3.12 are of Euler Cauchy type and hence these are transformed into equations with constant coefficients by the following substitutions (Ince 1956, Kamke 1959 and Kosko 1965) :

$$1+d'Z = .n \qquad \ldots. (4.3.14)$$

and

$$\eta = e^{v} \qquad \ldots . \ (4.3.15)$$

The final form of equation 4.3.11 is obtained as :

$$\frac{d^4 y'_{1u}}{dv^4} + 2\frac{d^3 y'_{1u}}{dv^3} + (\psi' - 1)\frac{d^2 y'_{1u}}{dv^2} + (\psi' - 2)\frac{dy'_{1u}}{dv} + \lambda = 0 \qquad \ldots . \ (4.3.16)$$

in which

$$\psi' = \frac{U}{(d')^2}$$

$$\ldots . \ (4.3.17)$$

$$\lambda = \frac{1}{(d')^4}$$

Substituting $Y'_{1u} = y'_{1u} + \frac{\lambda}{(\psi' - 2)} v$ in 4.3.16 leads to :

$$\frac{d^4 Y'_{1u}}{dv^4} + 2\frac{d^3 Y'_{1u}}{dv^4} + (\psi' - 1)\frac{d^2 Y'_{1u}}{dv^2} + (\psi' - 2)\frac{dY'_{1u}}{dv} = 0 \qquad \ldots . \ (4.3.18)$$

Equation 4.3.18 is a homogeneous ordinary linear differential equation with constant coefficients for which closed form solution can be arrived at, as follows :

Substituting $D \equiv \frac{d}{dv}$

$$[D^4 + 2D^3 + (\psi' - 1) D^2 + (\psi' - 2) D] Y'_{1u} = 0 \qquad \ldots . \ (4.3.19)$$

The solution is assumed to be of the form:

$$Y'_{1u} = oe^{\gamma v} \qquad \text{.... (4.3.20)}$$

Then the characteristic equation is given by

$$\gamma^4 + 2\gamma^3 + (\psi' - 1)\gamma^2 + (\psi' - 2)\gamma = 0 \qquad \text{.... (4.3.21)}$$

The 4 roots of the equation 4.3.21 are γ_1 to γ_4 :

$$\gamma_1 = 0$$

$$\gamma_2 = -1$$

$$\gamma_3 = \frac{-1 + \sqrt{9 - 4\psi'}}{2} \qquad \text{.... (4.3.22)}$$

$$\gamma_4 = \frac{-1 - \sqrt{9 - 4\psi'}}{2}$$

and the general solution is given by :

$$Y'_{1u} = Ae^{ov} + Be^{-v} + Ce^{\left(\frac{-1 + \sqrt{9-4\psi'}}{2}\right)v} + De^{\left(\frac{-1 - \sqrt{9-4\psi'}}{2}\right)v} \qquad \text{.... (4.3.23)}$$

Putting $\alpha = -\frac{1}{2}$ and $\beta = \frac{\sqrt{4\psi' - 9}}{2}$ then

$$\gamma_3, \gamma_4 = -\frac{1}{2} \pm \frac{i\sqrt{4\psi' - 9}}{2} = \alpha \pm i\beta \qquad \text{.... (4.3.24)}$$

and resubstituting $Y'_{1u} = y'_{1u} + \frac{\lambda}{(\psi' - 2)} \ln(1 + d'Z)$ into equation 4.3.23, it transforms to

$$y'_{1u} = \{A + \frac{B}{(1+d'Z)} + C(1+d'Z)^{\alpha} \operatorname{Cos}[\beta \ln(1+d'Z)]$$

$$+ D(1+d'Z)^{\alpha} \operatorname{Sin}[\beta \ln(1+d'Z)] - \frac{\lambda}{(\psi' - 2)} \ln(1+d'Z)\} \qquad \text{.. (4.3.25)}$$

Equation 4.3.25 represents the general solution for equation 4.3.16. For the lower layer the governing differential equation is :

$$\frac{d^2}{dZ^2}\left[(1+d'Z)^4 \frac{d^2y'_{11}}{dZ^2}\right] + U\frac{d}{dZ}\left[(1+d'Z)^2\frac{dy'_{11}}{dZ}\right] + y'_{11} = 0$$

$$Z_1 \leq Z \leq Z'$$

Substituting

$$(1+d'Z) = \eta$$

and $\eta = e^v$ and procoeding in the same way as before equation 4.3.12 transforms to :

$$\frac{d^4y'_{11}}{dv^4} + 2\frac{d^3y_{11}}{dv^3} + (\psi'-1)\frac{d^2y'_{11}}{dv^2} + (\psi'-2)\frac{dy'_{11}}{dv} + y'_{11} = 0 \quad \ldots. \ (4.3.26)$$

The general solution of equation 4.3.26 is similar to that of equation 3.3.23 and is given by (Ince 1956, Kamke 1959 and Kosko 1965):

$$y'_{11} = (1+d'Z)^{-\frac{1}{2}}\{A'\operatorname{Cosh}[\sigma'\ln(1+d'Z)]\ \operatorname{Cos}[\tau'\ln(1+d'Z)]$$
$$+ B'\operatorname{Cosh}[\sigma'\ln(1+d'Z)]\ \operatorname{Sin}[\tau'\ln(1+d'Z)] + C'\operatorname{Sinh}[\sigma'\ln(1+d'Z)]$$
$$\operatorname{Cos}[\tau'\ln(1+d'Z)] + D'\operatorname{Sinh}[\sigma'\ln(1+d'Z)]\ \operatorname{Sin}[\tau'\ln(1+d'Z)]\} \quad \ldots (4.3.27)$$

The arbitrary constants A,B,C,D, A', B',C' and D' alongwith the unknown load, Q_{tD} or M_{tD}, such that the soil goes into plastic region upto a depth of L_1 can be found from the four boundary conditions at the tip and top of the pile and the five compatibility

conditions at the interface.

For the case (i) i.e. for free-free pile acted upon by axial load P and later force Q_t, the boundary conditions are :

I at $x = 0$ i.e. at $Z = 0$

a) $B.M = 0$

$$EI(x) \frac{d^2 y_u}{dx^2} = 0 \qquad \text{.... (4.3.28 a)}$$

or

$$\frac{d^2 y_u}{dZ^2} = 0 \qquad \text{.... (4.3.28)}$$

and

b) $S.F = -Q_t$

$$\frac{d}{dx}\left[-EI(x) \frac{d^2 y_u}{dx^2}\right] - P(x) \frac{dy_u}{dx} = -Q_t \qquad \text{....(4.3.29a)}$$

or

$$\frac{d^3 y'_{1u}}{dZ^3} + 4d' \frac{d^2 y'_{1u}}{dZ^2} + U \frac{dy'_{1u}}{dZ} = \frac{Qt}{tR} = Q_{tD} \qquad \text{.... (4.3.29)}$$

where Q_{tD} = Non-Dimensional shear force at the top

II at $x = L_1$ or $Z = Z_1$ the five compatibility conditions are :

a) Deflection compatibility :

$$y'_{1u} = y'_{11} \qquad \text{....(4.3.30)}$$

b) Slope compatibility:

$$\frac{dy'_{1u}}{dZ} = \frac{dy'_{1l}}{dZ} \qquad \ldots. (4.3.31)$$

c) Bending moment compatibility :

$$\frac{d^2 y'_{1u}}{dZ^2} = \frac{d^2 y'_{1l}}{dZ^2} \qquad \ldots. (4.3.32)$$

d) Shear compatibility :

$$\frac{d^3 y'_{1u}}{dZ^3} = \frac{d^3 y'_{1l}}{dZ^3} \qquad \ldots. (4.3.33)$$

and

e) Pressure compatibility :

$$y'_{1l} = 1 \qquad \ldots. (4.3.34)$$

III at $x = L$ or $Z = Z'$

a) B.M. = 0

$$\frac{d^2 y'_{1l}}{dZ^2} = 0 \qquad \ldots. (4.3.35)$$

and

b) S.F = 0

$$\frac{d^3 y'_{1l}}{dZ^3} + \frac{4d'}{(1+d'Z')}\frac{d^2 y'_{1l}}{dZ^2} + \frac{U}{(1+d'Z')^2}\frac{dy'_{1l}}{dZ} = 0 \qquad \ldots. (4.3.36)$$

By using the above .9 boundary conditions (equations 4.3.27 to 4.3.35) the constants A, B, C, D, A', B', C' and D' and the lateral load Q_{tD} required for a particular value of Z_1 to go into plastic resistance are evaluated.

For the case (ii) i.e. free-free pile acted upon by a moment M_t and an axial load P and case (iii) fixed-free pile the general form of the solutions, remain same. The boundary conditions at x = L and compatibility conditions at $x = L_1$ also remain same, whereas the boundary conditions at the top of the pile, at x = 0 are :

Case ii) For free-free pile with only P and M_t acting at top,

at x = 0.

a) B.M. = M_t .... (4.3.37a)

or

$$\frac{d^2 y'_{2u}}{dZ^2} = \frac{M_t}{tR^2} = M_{tD} \qquad \text{.... (4.3.37)}$$

where M_{tD} = Non-dimensional moment.

and b) S.F = 0

$$\frac{d^3 y'_{2u}}{dZ^3} + 4d' \frac{d^2 y'_{2u}}{dZ^2} + U \frac{dy'_{2u}}{dZ} = 0 \qquad \text{.... (4.3.38)}$$

and for case (iii) Fixed-free end conditions pile with only P and Q_t acting at top

at x = 0

a) slope = 0

i.e. $$\frac{dy'_{1f}}{dZ} = 0. \qquad \text{.... (4.3.39)}$$

b) $S.F = -Q_t$

$$\frac{d^3 y'_{1f}}{dZ^3} + 4d' \frac{d^2 y'_{1f}}{dZ^2} + U \frac{dy'_{1f}}{dZ} = \frac{Q_t}{tR} = Q_{1f} \qquad \text{.... (4.3.40)}$$

where Q_{1f} = Non-dimensional shear force at the top.

4.4 RESULTS AND DISCUSSION.

Numerical results are obtained for normal ranges of variables, based on the generalized solutions presented in the previous section. Here again Z' and c are taken as 4.0 and 0.6 respectively. Results are obtained for $\frac{U}{U_{cr}}$ = 0.3 and 0.6 wherein U_{cr} = 0.9193. As most probably only a small portion near the ground surface may go into plastic region, the value of Z_1 is likely to be around 0.25 to 0.5. Hence numerical results are obtained for Z_1 = 0.2, 0.4 and 0.6 which covers a wide range of practical interests.

Figs. 4.3 and 4.4 show the effect of Z_1 on Q_{tD} and M_{tD} respectively for $\frac{U}{U_{cr}}$ = 0.3 and 0.6. It is seen from these that, for $\frac{U}{U_{cr}}$ = 0.3, as Z_1 increases from 0.2 to 0.6 the lateral force required Q_{tD} or M_{tD}, for a depth Z_1 of the soil at the top to go into plastic region, also increases considerably. Hence as expected in this case as the lateral load is increased more and more soil at the top goes into plastic zone. Whereas for $\frac{U}{U_{cr}}$ = 0.6 even for a small increment in the lateral load the depth upto which the soil at the top goes into plastic region considerably increases. This is because

at higher axial loads on the pile, the soil at the top does not take much lateral load to reach ultimate resistance.

Fig. 4.5 shows the effect of Z_1 on the deflection coefficient at $Z = 0$ for $\frac{U}{U_{cr}} = 0.3$ and 0.6 . Here it is seen that as the depth of plastic zone increases the deflection coefficient y_1' at $Z = 0$ increases considerably for both $\frac{U}{U_{cr}} = 0.3$ and 0.6. Fig. 4.6 depicts the effect of Z_1 on y_2' at $Z=0$ for $U/U_{cr} = 0.3$ and 0.6 . Here again the same trend is seen as in Fig. 4.5.

Fig. 4.7 shows the relationship between the depth of plastic zone, Z_1, and the corresponding lateral load required, Q_{1f}, for fixed head pile for $U/U_{cr} = 0.3$ and 0.6. Here again as expected the lateral load required at the top increases with increasing Z_1.

Fig. 4.8 shows the effect of Z_1 on fixity moment at the fixed end of the pile. It is seen that as Z_1 increases the moment at the fixed end increases as can be expected.

Fig. 4.9 shows the effect of increasing Z_1 on the deflection coefficient at top, $\bar{y}_{1f}'$, for a fixed head pile for $U/U_{cr} = 0.3$ and 0.6. It is seen that the effect of $\bar{U}/U_{cr}$ is considerably less on the deflection coefficient for fixed head pile. It is also seen when compared to free-free pile the deflection and moment coefficient values are considerably less for fixed-free pile for the same value of Z_1 and lateral load.

4.5 CONCLUSIONS

As stated earlier the flexural behaviour of axially and laterally loaded piles can be reasonably carried out at working loads on the concept of soil modulus. However, with increasing lateral loads the soil in the top region reaches its ultimate resistance value and as such for any realistic analysis one has to take into account the elasto-plastic nature of the soil. With this in view the analysis presented in this Chapter can be used for analysing the problem of axially and laterally loaded piles with variable cross-section, when the pile is subjected to lateral loads which are greater than working loads. Also the analysis brings out the fact that by assuming the soil to be elasto-plastic and analysing the problem, the effect of non-linearity between load-deflection curves observed in the field can be accounted for.

The results obtained show that the boundary conditions at the top depth of plastic zone, and the axial force coming have an overwhelming influence on the flexural behaviour.

The solutions being obtained in closed form, these can be also useful for checking the numerical efficiency of other approximate methods which are in general used for solving such complicated problems.

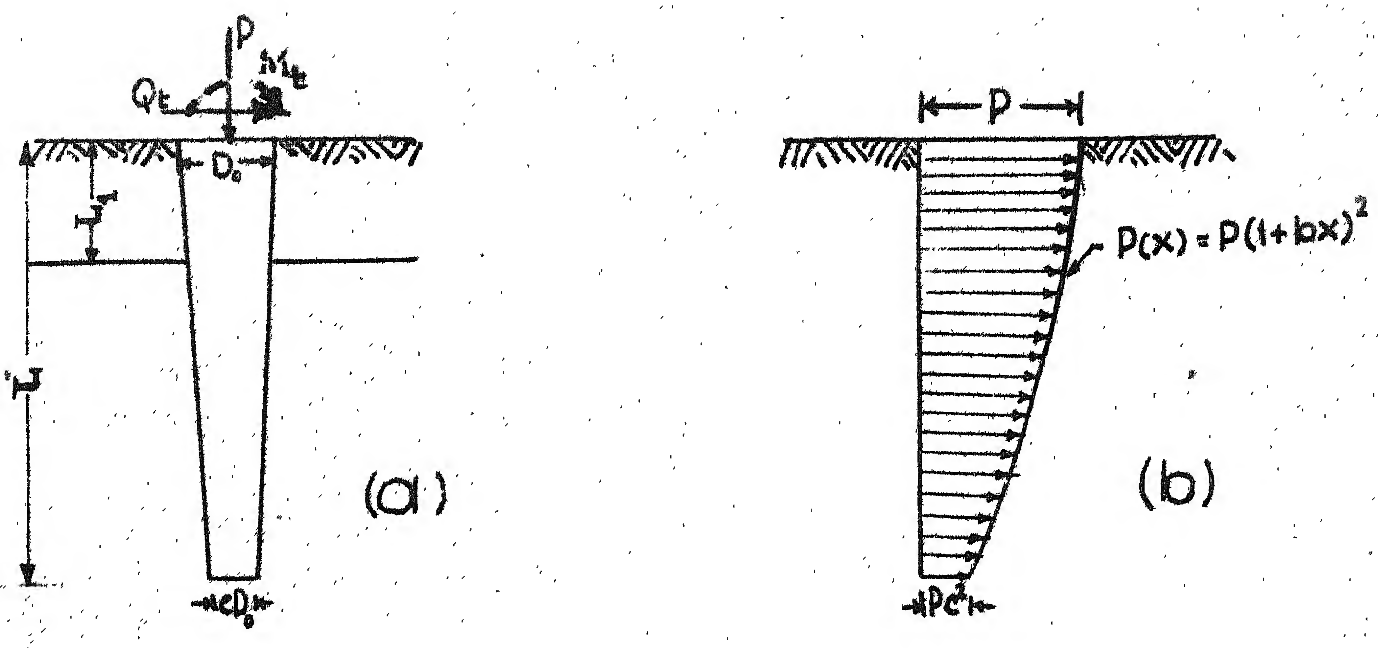


FIG. 4-1. TAPERED PILE UNDER THE ACTION OF LATERAL AND AXIAL LOADS IN ELASTO-PLASTIC SOIL.

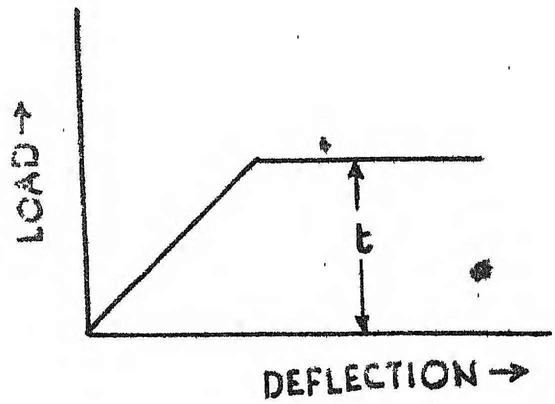


FIG. 4-2. LOAD-DEFLECTION RELATIONSHIP

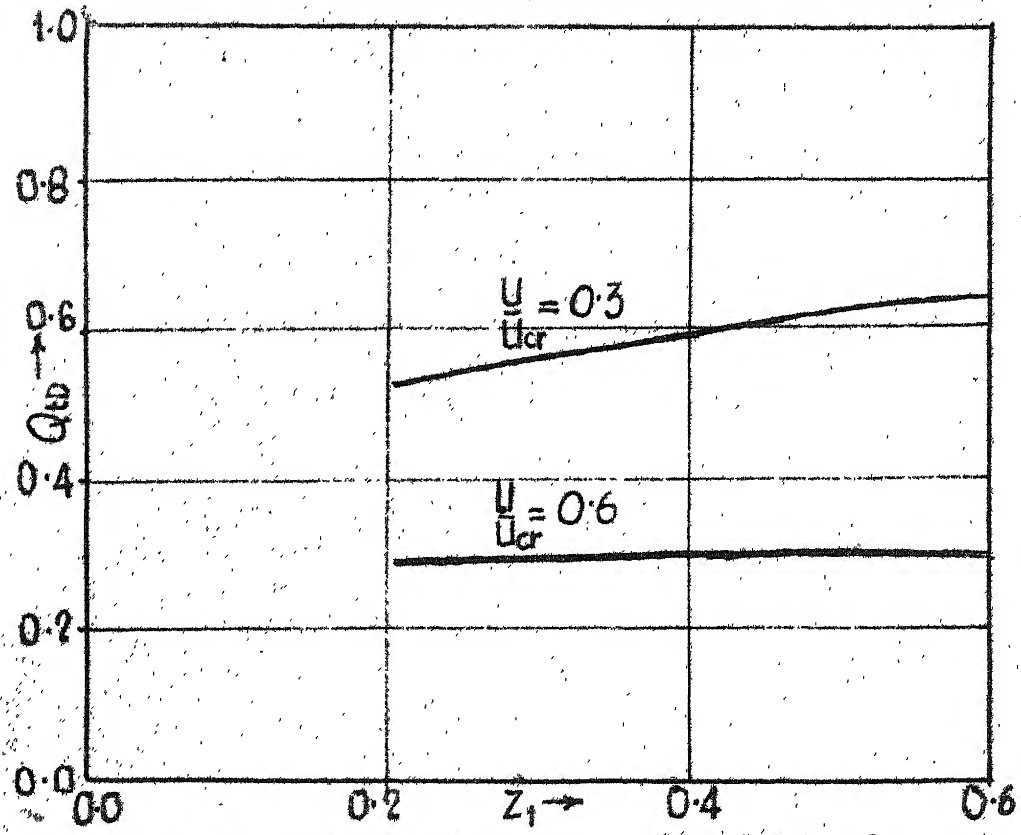


FIG. 4-3 EFFECT OF Z_1 ON Q_{tD}

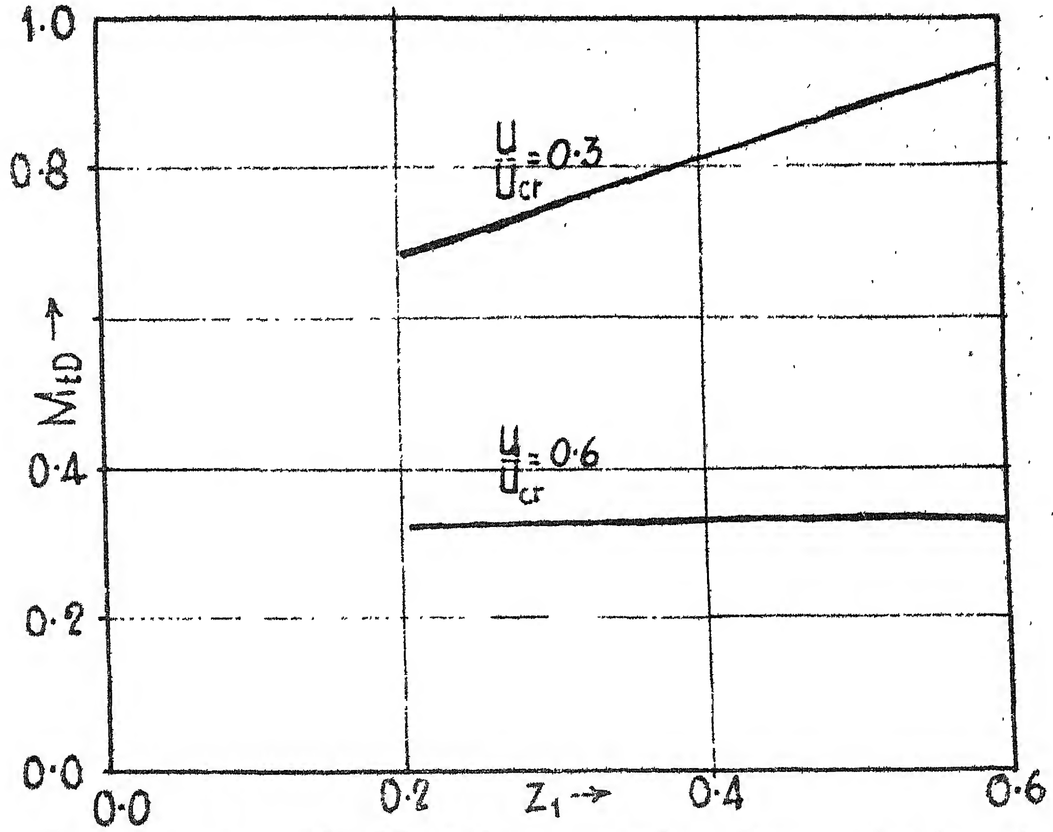


FIG. 4-4 EFFECT OF Z_1 ON M_{tD}

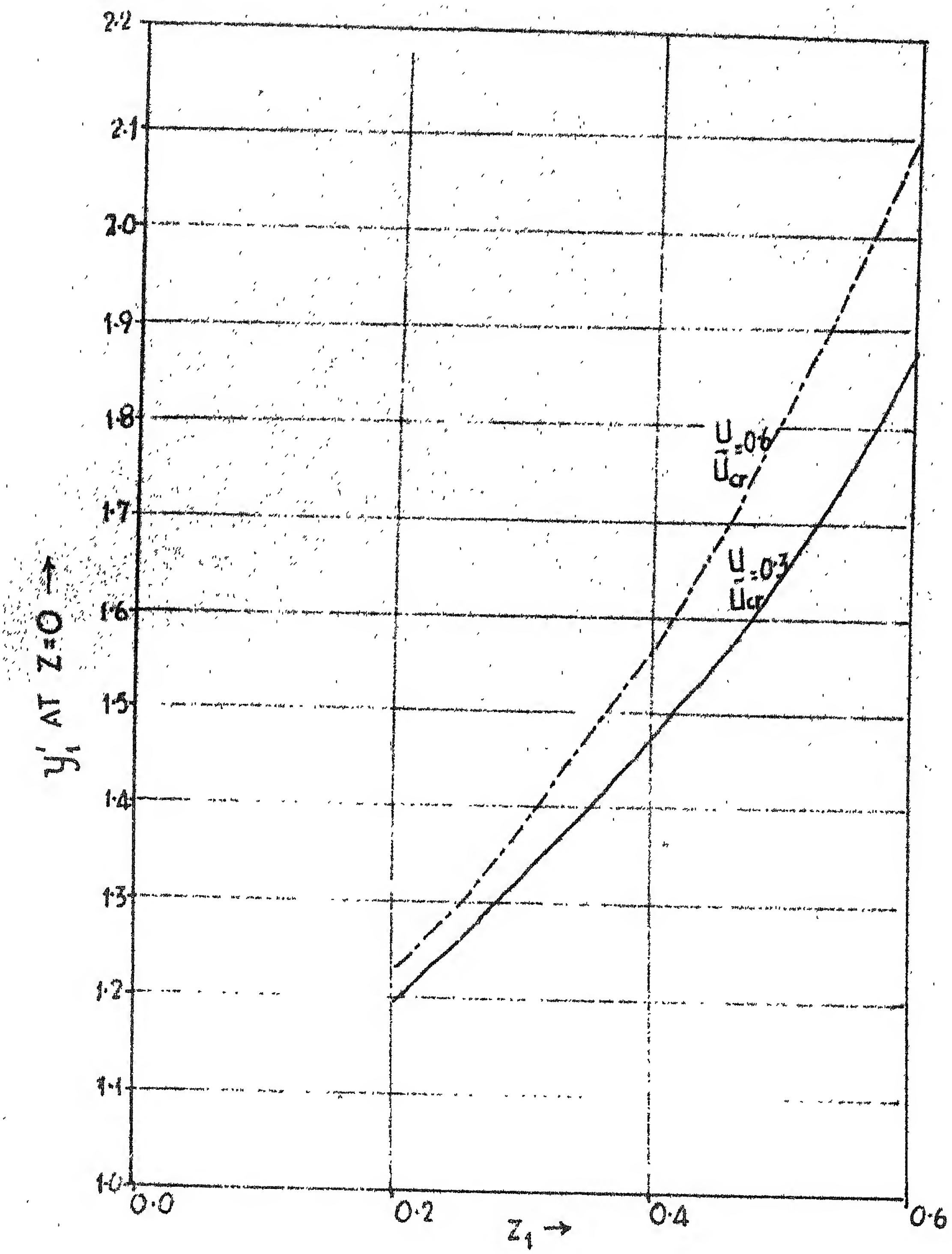


FIG. 4.5 EFFECT OF Z_1 ON y_1' AT Z=O

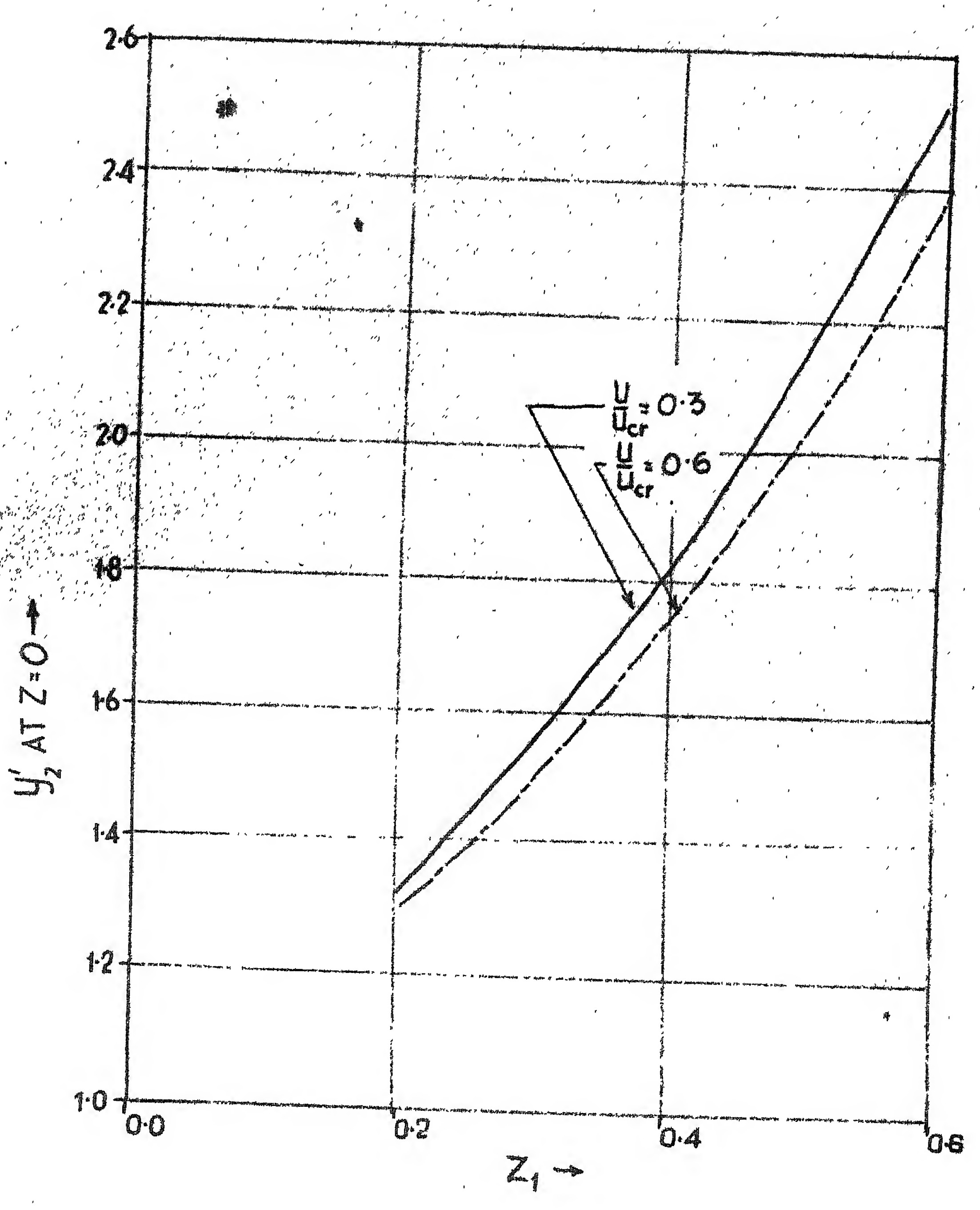


FIG. 4·6 EFFECT OF Z_1 ON y_2' AT Z=O

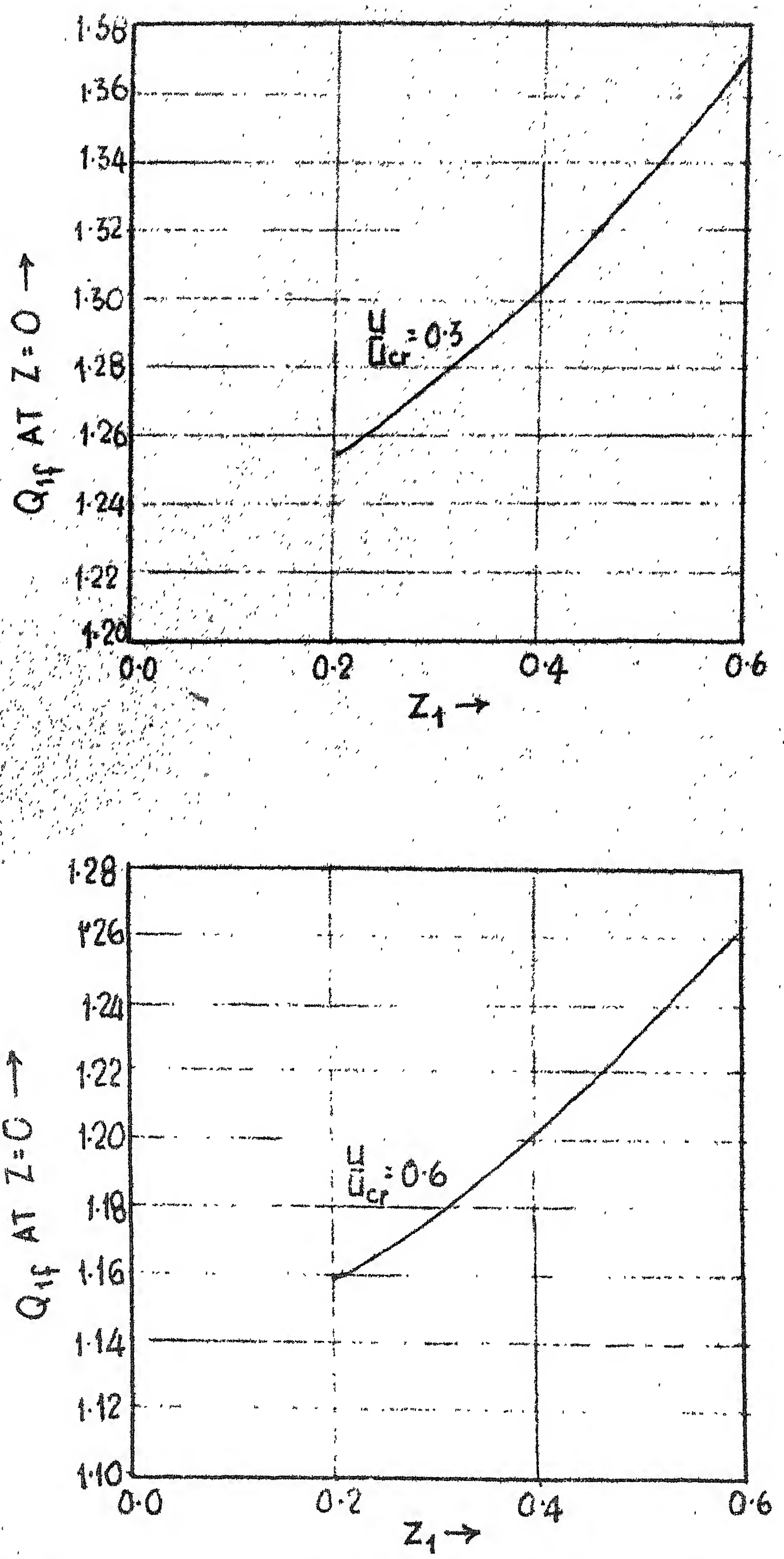


FIG. 4.7 EFFECT OF Z_1 ON Q_{1f}

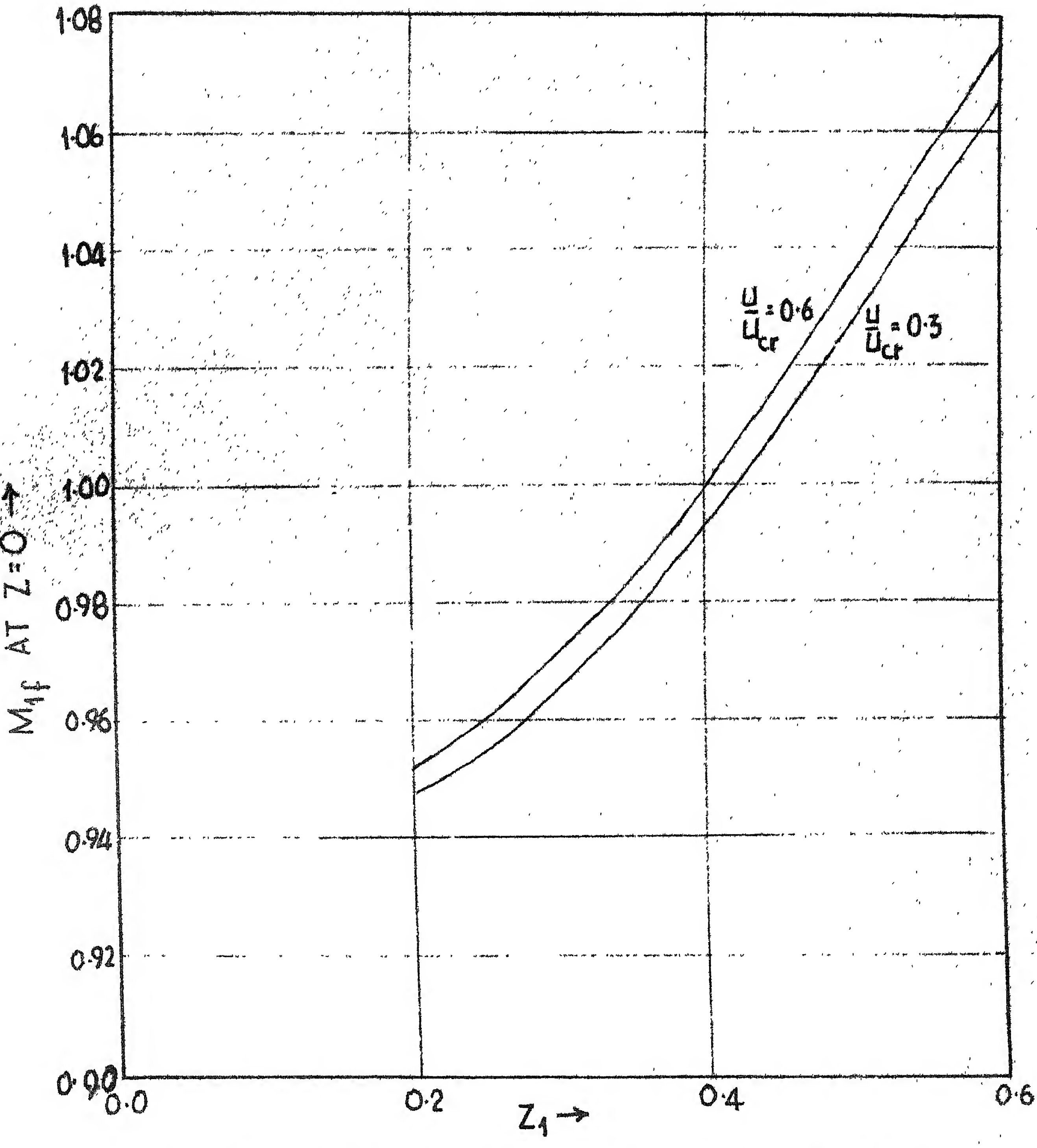


FIG. 4·8 EFFECT OF Z_1 ON M_{1f} AT $Z=0$.

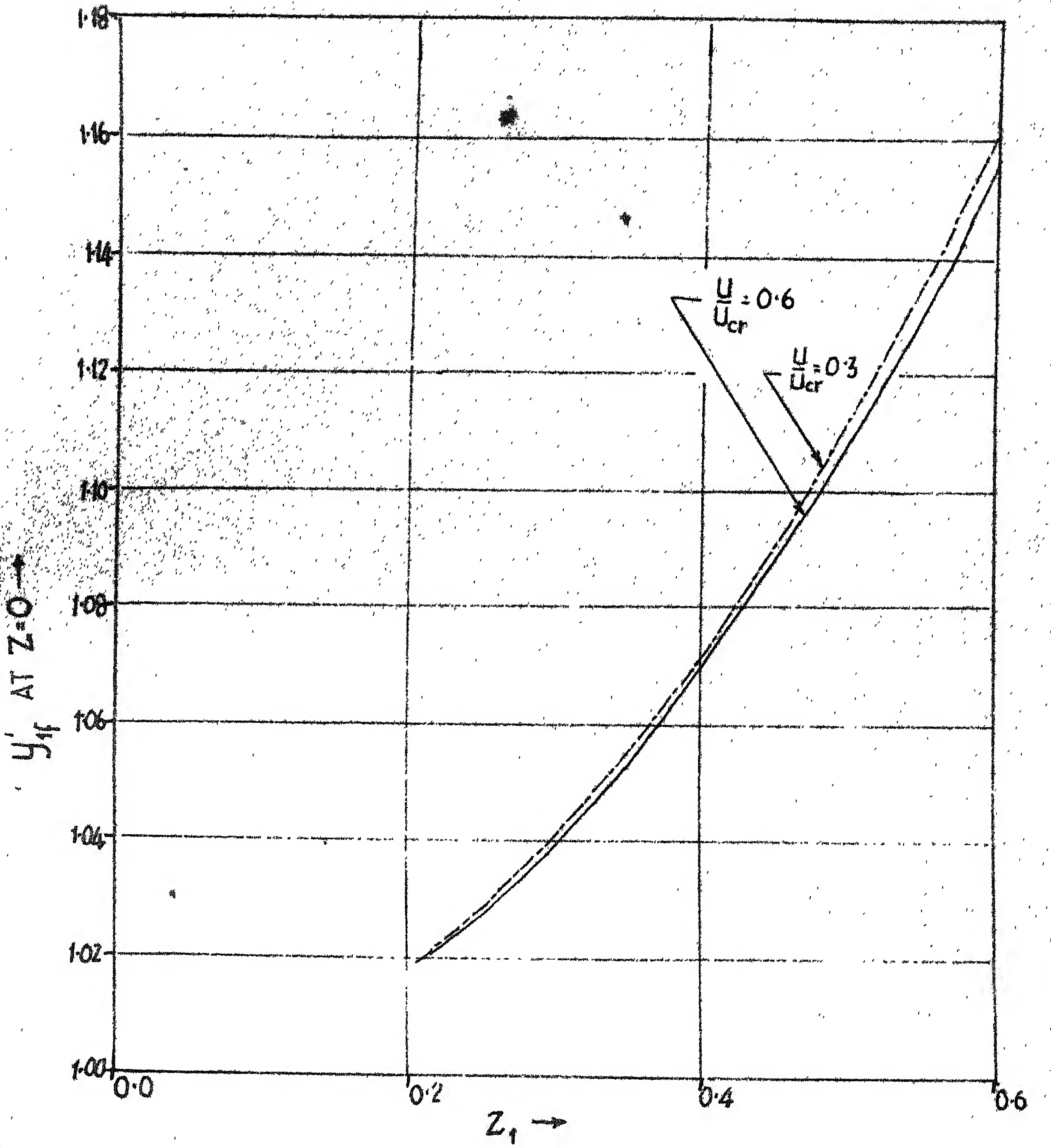


FIG. 4.9 EFFECT OF Z_1 ON y'_{1f} AT Z=0.

CHAPTER 5

PROBABILISTIC APPROACH TO DESIGN OF PILES.

5.1 INTRODUCTION

In the previous chapters it was considered that the soil properties, pile properties and the loads coming on the pile are deterministic in nature. However, in reality they are random in nature with certain control in supervision. The necessity for probabilistic considerations in soil mechanics arises from the fact that soil is not a material made to satisfy a given specification, like steel and concrete, but a natural product of which the properties may vary from place to place considerably. This variation may be due to the uncertainties in the soil properties or due to stratifications that contain several distinct layers. The probabilistic analysis of the latter is relatively involved and as such much work has not been done taking this factor into account. However, many authors (Lazard 1961, Langejan 1965, Lumb 1966, Wu and Kraft 1967, Resendiz and Herrera 1969, Shuk 1969, Biernatowsky 1969, Wu and Kraft 1970 and Benjamin 1970) have considered the randomness in the soil properties in analysing problems of foundation engineering and soil mechanics ignoring the spatial correlation.

Analysis of the random variation of the strength properties of materials such as concrete and steel by many authors

(Freudenthal 1947, 1956, 1961 ,Lumb 1966 and others) show that a number of probability distributions can be used to describe the variation. These include the normal, log-normal and extreme value distributions. Since the controlling factors of the distributions are not very well known, it is necessary to determine the appropriate probability function empirically by fitting to the available experimental data.

Eventhough in recent times few studies have been reported wherein probabilistic approach is used for the design of foundations, there is paucity of literature regarding the use of this technique for the design of pile foundation. As the design of piles is done on understrength and overload factor basis (Broms 1964), the concept of these factors of safety from probabilistic approach is considered at some length in this chapter. The effect of distribution function, 'Large sample theory' and 'small sample theory' (Alder and Roessler, 1961) as affecting the factor of safety is brought out.

5.2 REVIEW OF LITERATURE

As stated earlier, there is not much literature available regarding the application of probabilistic approach specifically for the design of pile foundations. With this in view a brief review of the existing literature on probabilistic approaches to soil mechanics problems in general is presented below.

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classical probabilistic approach to structural problems, only recently, Lazard (1961) presented a discussion on probabilistic approach to soil mechanics problem very briefly.

Langejan (1965) analysed the problem of the stability of a slope wherein .. it is assumed that the governing soil property is only the shear strength, S_f, and is taken to be a random variable. The design shear strength, S_d, is taken as equal to $\bar{S}_f/F$ where F is the factor of safety applied. The probability of failure is stated as

$P\left[S_{ft} < \frac{\bar{S}_f}{F}\right]$ wherein S_{ft} = true shear strength of the soil : .

and $\bar{S}_f$ = mean shear strength, evaluated from the laboratory and/ or field tests.

Further, it is assumed that the distribution of S_f to be log-normal and the probability of failure of the slope for different values of factor of safety, F, variance of S_f, s, and the number of samples, N . It is seen from the results presented that variance and number of samples tested have overwhelming influence on the failure probability.

Langejan (1965) further studied the economic aspects of the failure of an excavation and arrived at an optimum slope based on the statistical-economic decision theory.

Lumb (1966) presented the results of tests conducted on four different types of soils from Hong Kong, Viz.: (a) soft marine clay deposited in shallow waters (b) alluvial sandy clay (c) a recidual silty sand (d) a recidual clayey silt. The properties studied

include Atterberg limits, grading and for undisturbed samples, strength and compressibility characteristics. Herein it is shown that the properties follow Gaussian distribution or related to Gaussian distribution. Comparison of the observed and theoretical values by χ^2 test show good aggreement between them. In the analysis only uncertainty in the soil properties is considered.

Wu and Kraft (1967) considered the randomness in both soil properties and loads in the analysis of foundation safety. The experimental data used in the analysis was obtained from a total of about twenty five borings in an area of about 15000 sq.ft. and about 100 unconfined tests were made on 2-inch diameter tube samples. The site consists of a brown sandy clay till and a grey clay till with lenses of fine to medium sand. The results of the above tests fit the log-normal distribution fairly well. The results of the standard penetration tests conducted on sands in 12 borings also fit the log-normal distribution reasonably good.

Wu and Craft (1967) further verified Terzaghi and Peck's (1948) recommendation which says that, the measured strength with in a significant depth from each boring be averaged and the smallest average be used for design purposes. It is seen that if the minimum average strength is used for the design then the variations in soil properties and the size of failure area do not exert a predominant influence on safety. In the above calculations, the dead load is assumed to be deterministic and the live load is assumed to be

normally distributed, and the wind load to be exponentially distributed. It is also shown that an increase in number of borings in a given site improves the probability of safety considerably whereas an increase in number of samples in each boring does not have much effect on the probability of safety.

Kuhn and Burton (1967) proposed a statistical method to control the "dry density for a road construction." In the proposed sampling scheme a method is outlined whereby engineering dicisions can be taken on the number of samples to be tested to satisfy various engineering requirements such as the error in the test results and the desired limits of accuracy for a specified probability. It is seem from the test results presented that the dry density fits the normal distribution fairly well.

Resendiz and Herrera (1969) studied the settlement and rotation of rectangular foundations on randomly compressible soils for both flexible and rigid foundations . Herein all variations in compressibility occuring in theh rizontal direction are random. It is shown that the average settlement and rotation of a foundation can be regarded as normally distributed also other parameters being equal, the probability of rotation exceeding a certain value increases with the rigidity of the foundation.

Shuk (1969) presented an analysis of variability of natural soils in the design of footings with regard to settlements due to

consolidation. The total expected building cost, which is the sum of initial cost plus the expected maintainance cost, for the conditional probability of exceeding the given maximum allowable total or differen - tial settlement is plotted against the probability of exceeding a maximum allowable settlement. The probability corresponding to the minimum total expected cost gives the "Optimum design probability".

Biernatowski (1969) made an analysis of stability of slopes with probabilistic considerations. In the analysis all the parameters are taken to be random variables except the geometrical dimensions. Analysis is presented for both normal and log-normal distribution of random variables and for different degrees of slope.

Benjamin (1970) for the first time used Bayesian decision theory approach in predicting the probable settlements of foundations. Herein an apriori distribution is assumed before the collection of data based on the experience and professional judgement. After filling the data, Posterior distribution is obtained by using Bayesian theory. The total expected loss which is the sum of sampling cost and expected terminal loss, is minimized to get the optimal decision.

Wu and Kraft (1970) presented the analysis for the probability of failure of slopes. For various values of conventional factor of safety the failure probability has been evaluated and statistical decision theory is applied to the selection of optimum slope. Here again the optimum design is one which minimizes the expected cost or

maximizes the expected utility with in the economic or physical constraints imposed by the problem. It is shown in this work that the optimum safety factor is not unique quantity but depends on the stability number of the slope.

Kogan and Lupasko (1970) presented the variational method of evaluating slope stability at limiting equilibrium conditions. Here a general shape of sliding surface is assumed, whereas earlier works were confined to circular arcs, logarithmic spirals, etc.

Ignatov (1970) used the method of end differences to solve the problem of designing rectangular slabs resting on a statistically non-homogeneous base and subjected to uniformly distributed loading applied over rectangular areas.

5.3 ANALYSIS

As stated earlier in general the design of laterally loaded piles is carried on the basis of over-load and under-strength factors (Broms, 1964a, 1964 b) . Thus the failure of the pile can take place either when (i) the actual loads coming onto the structure exceed considerably than those used while designing and/or (ii) if the strength parameters of the soil has been over estimated. Soil being a natural product has considerable variation and as such one should really think in terms of probabilistic variation of strength while dealing with the design of laterally loaded piles at ultimate

resistance. Having recognized the fact that the strength should be taken as a random function with a certain probabilistic distribution based on the experimental data, one can then consider the partial factor of safety in terms of uncertainties. In what follows, it is assumed that all other factors are deterministic whereas only the properties of the soil are considered with uncertainty. However, for a complete analysis one has to consider the uncertainties both in loadings as well as in properties of the soil and pile material. Also, it should be noted that the following discussion deals only with the factor of safety concept and does not give complete analysis of the problem of design of piles for resisting lateral loads.

For a short, rigid pile in a cohesive soil, it is assumed that the failure takes place, when all along the embedded length of the pile – 1·5 d the soil reaches the ultimate resistance which is equal to $9c_u d$ where $c_u = \frac{1}{2}$ unconfined compressive strength, and d = diameter of the pile.

Let the shear strength S_f be the random variable, with certain distribution. Then in the conventional method, the design shear strength, S_d is given by :

$$S_d = \bar{S}_f / F$$

in which $\bar{S}_f$ = mean value of the shear strength obtained from laboratory and/or field tests.

and F = Conventional factor of safety.

Now taking uncertainties in soil strength, the probability of soil reaching ultimate strength can be expressed as follows :

$$P_f = P\left[\left(S_{ft} - \frac{\bar{S}_f}{F} \right) < 0 \right] \qquad \dots\dots (5.1)$$

in which P_f = the probability of soil reaching ultimate resistance

and S_{ft} = True in - situ shear strength

If the number of samples tested from a population are less than 30, then small sample theory, given by Alder and Roessler (1961) has to be used, wherein Student's t-distribution is used to approximate the actual distribution of S_f.

Case (i) If S_f is normally distributed, then

$$t = \frac{\bar{S}_f - S_{ft}}{s}\sqrt{N} \qquad \dots\dots (5.2)$$

in which $s = \sqrt{\dfrac{\Sigma(\bar{S}_f - S_f)^2}{N-1}}$, where N = number of samples tested.

t = the variable of Student's distribution function.

Substituting for S_{ft} from equation 5.2 into equation 5.1 , it reduces to :

$$P_f = P\left[t > \frac{\bar{S}_f}{s} \left(\frac{F-1}{F} \right) \sqrt{N} \right] \qquad \dots\dots (5.3)$$

In a given case the values of $\bar{S}_f$, s, F and N are known ; then P_f can be obtained from tables of the t-distribution found in many statistical handbooks (Fisher and Yates, 1957).

TABLE I. Values of P_f for $\bar{S}_f = 0.8$ Kgs./cm^2 and for different values of Number of Tests (N), Standard Deviation (s), and Factor of Safety (F).

N	F = 1.2			F = 1.5			F = 2.0		
	s=0.10	s=0.05	s=0.02	s = 0.10	s = 0.05	s=0.02	s=0.10	s=0.05	s=0.02
2	0.1562	0.0899	0.0379	0.0899	0.0452	0.0200	0.0602	0.0305	0.0128
3	0.0300	0.0232	0.0043	0.0232	0.0062	0.0019	0.0120	0.0042	< 0.0005
5	0.0219	0.0035	< 0.0005	0.0035	< 0.0005	< 0.0005	< 0.0005	< 0.0005	< 0.0005
10	0.0022	< 0.0005	< 0.0005	< 0.0005	< 0.0005	< 0.0005	< 0.0005	< 0.0005	< 0.0005

S_f normally distributed

$N < 30$

Table I gives the magnitude of P_f, calculated from values of N, s, and F that represent the range of practical possibilities. Herein $\overline{S}_f$ is assumed to be 0.8 Kgs./sq.cm. From this table it appears, that if a pile is designed with a given value of the safety factor F, the probability of soil reaching ultimate resistance may vary within wide range of values. Also it is seen that there is a sharp increase in P_f for small values of N.

Case (2) If S_f is log-normally distributed :

Now the $P_f = P\left[S_{ft} - \frac{\overline{S}_f}{F} < 0 \right]$ can be written as

$$P_f = P\left[S_{ft} F < \overline{S}_f \right] \qquad \ldots. (5.4)$$

Since the quantities on either side are not negative, taking logarithm on both sides it transforms to :

$$P_f = P\left[\ln(S_{ft}F) < \ln(\overline{S}_f) \right] \qquad \ldots. (5.5)$$

It is well known that, $A.M \geq G.M$ .... (5.6)

in which A.M = arithmatic mean of a set of numbers.

G.M = Geometric mean of a set of numbers.

If $S_{f1}, S_{f2}, \ldots. S_{fN}$ are the possible values of S_f then from equation (5.6)

$$\frac{(S_{f1} + S_{f2} + \ldots. + S_{fN})}{N} \geq (S_{f1} . S_{f2} . S_{f3} \ldots . S_{fN})^{1/N} \qquad \ldots. (5.7)$$

Equality holds good only if $S_{f1} = S_{f2} = \ldots = S_{fN}$

$$\ln(\bar{S}_f) \geq \frac{1}{N} \ln \left(\prod_{i=1}^{N} S_{fi} \right) \qquad \ldots.(5.8)$$

i.e.

$$\ln(\bar{S}_f) \geq \overline{\ln(S_f)} \qquad \ldots.(5.9)$$

From equation 5.5 and equation 5.9 it is seen that only a lower bound can be given :

$$P_f = P\,[\ln(S_{ft}\,F) < \ln(\bar{S}_f)\,]$$

$$\geq P[\,\ln(S_{ft}F) < \overline{\ln(S_f)}\,] \qquad \ldots.\ (5.10)$$

Again by small sample theory

$$t = \left(\frac{\overline{\ln S_f} - \ln S_{ft}}{s} \right) \sqrt{N} \qquad \ldots.\ (5.11)$$

in which

$$s = \sqrt{\sum_{i=1}^{N} \frac{(\overline{\ln S_f} - \ln S_{fi})^2}{N-1}}$$

Now substituting for $\ln(S_{ft})$ from equation 5.11 into equation 5.10 P_f is obtained as :

$$P_f \geq P\,[t > \frac{\sqrt{N}}{s} \ln F\,] \qquad \ldots.\ (5.12)$$

For different values of N, s and F, the lower bound for P_f is evaluated.

TABLE II. Lower bound for P_f for $\overline{S}_f = 0.8$ Kgs/cm^2 and for different values of number of tests(N), Standard Deviation (s) and factor of safety (F).

S_f log-normally distributed.

N	F = 1.2			F = 1.5			F = 2.0		
	s = 0.1	s = 0.05	s=0.02	s=0.1	s=0.05	s=0.02	s=0.1	s=0.05	s=0.02
2	0.1214	0.0677	0.0245	0.0515	0.0295	0.0125	0.0392	0.0196	0.0074
3	0.0436	0.0134	0.0023	0.0100	0.0034	-	0.0048	0.0021	-
5	0.0078	0.0009	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-

\- Denotes that the lower bound for P_f for these values of N, s, and F cannot be evaluated.

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Table II represents P_f values for different values of N, s and F. Here again it is seen that for a given factor of safety the probability of failure varies widely. It is also seen from the Tables I and II that the P_f is considerably less for log-normal distribution of S_f when compared to that of normal distribution of S_f, for the same values of N, s and F.

Usually a sample from a population is said to be 'Large' if the number of samples tested is greater than 30. However it should be noted that there is no clear cut boundary between 'Large Sample' and 'Small Sample'.

According to central limit theorem "if X has a distribution with mean, m, and Standard deviation, σ, and if all possible samples of N variates are drawn then the sample mean, $\bar{X}$, will have a distribution which for larger and larger N, approaches the normal distribution with mean, m and Standard deviation $\sigma / \sqrt{N}$" (Alder and Roessler, 1961).

i.e. $$Z = \frac{\bar{X} - m}{\sigma_{\bar{X}}} \sim N(0,1) \text{ for large } N \qquad \ldots(5.13)$$

where $\sigma_{\bar{X}} = \sigma / \sqrt{N}$.

So it is seen from this theorem that whatever may be the distribution of S_f the sample mean $\bar{S}_f$ will be normally distributed for large N. With this in view a general equation is given below for P_f :

$$P_f = P\,[S_{ft} < \frac{\overline{S}_f}{F}] \qquad \ldots. (5.14)$$

but according to the above theorem

$$Z = \frac{(\overline{S}_f - S_{ft})}{s}\sqrt{N} \sim N(0,1) \quad \text{for large N.} \qquad \ldots..(5.15)$$

$$\text{i.e.}\quad S_{ft} = -\frac{sZ}{\sqrt{N}} + \overline{S}_f \qquad \ldots. (5.16)$$

Substituting for S_{ft} in equation 5.14 from equation 5.16, it is seen that,

$$P_f = P\,[Z > \frac{\sqrt{N}}{s}\ \overline{S}_f(\frac{F-1}{F})\ ] \qquad \ldots.(5.17)$$

Using equation 5.17 for different values of N, s and F , P_f is caloulated for $\overline{S}_f$ = 0.8 Kgs/cm^2 and is presented in Table III. Here again it is seen that the effect of N, s and F is quite considerable.

TABLE III. Values of P_f for $\bar{S}_f \approx 0.8$ Kgs/cm^2 and for different values of Number of tests (N), Standard Deviation (s) and Factor of Safety (F).

N	F = 1.2			F = 1.4			F = 1.6		
	s=0.30	s=0.25	s=0.20	s=0.30	s=0.25	s=0.20	s=0.30	s=0.25	s=0.20
30	0.00755	0.00175	0.00014	1.5×10^{-6}	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$
35	0.00427	0.00084	0.00004	3.4×10^{-6}	$<10^{-6}$	$<10^{-6}$	"	"	"
40	0.00248	0.00038	0.00001	0.7×10^{-6}	$<10^{-6}$	$<10^{-6}$	"	"	"
45	0.00144	0.00017	4.1×10^{-6}	$<10^{-6}$	$<10^{-6}$	$<10^{-6}$	"	"	"

5.4 SUGGESTIONS FOR FURTHER RESEARCH

Based on the review of literature and the above analysis certain suggestions for further research are made . The above analysis clearly brings out the fact that in the actual design it is worthwhile to consider the probabilistic variation of soil properties which would give the probability of failure rather than using arbitrary factors of safety. Also the analysis shows the effect of distribution functions, large and small sample theory as affecting the probabilistic analysis.

However for a realistic analysis, the loads and pile properties are also to be considered as probabilistic rather than deterministic. Also regression analysis is to be used to consider the spatial correlation. The actual distribution function of the soil property should be found out by fitting empirically to the experimental data available. In the absence of data subjective probabilities have to be used in making decisions. With all this in view, it is seen that, there is ample scope for further research in probabilistic approach to soil mechanics problems.

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